MVA2000 IAPR Workshop on Machine Vision Applications, Nov. 28–30, 2000, The University of Tokyo, Japan

13—10

Nonlinear Diffusion of Normals for Stable Detection of Ridges and Ravines on Range Images and Polygonal Models

Yutaka Ohtake and Alexander Belyaev *
The University of Aizu

Abstract

The main contribution of this work consists of developing a new surface smoothing method. The method preserves sharp variation points of the surface normals and is good for stable extraction of perceptually salient ridges and ravines. The method is based on a discrete nonlinear diffusion process (iterative nonlinear averaging) applied to the surface normals. We demonstrate applicability of the method to analysis and segmentation of range images (height fields) and polygonal surfaces.

1 Introduction

Shape features invariant under rotations, translations, and scalings are important for studying shapes of 3D objects. The paper presents a novel smoothing method for stable detection of surface creases, ridges and ravines, on range images and polygonal surfaces (triangle meshes). View-independent ridges and ravines can be intuitively defined as curves on a surface along which the surface bends sharply or, equivalently, sharp variation points of the surface normals.

The above definition of the ridges and ravines resembles a definition for edges in image processing as sharp variation points of an image intensity. In [2, 3, 4] we explored this analogy between the edges of intensity images and ridges and ravines of smooth surfaces and extended edge detection methods to ridge and ravines extraction on range images and triangular surfaces.

The main contribution of this paper consists of developing a new smoothing method. The method preserves sharp variation points of the surface normals and is good for stable extraction of perceptually salient ridges and ravines. We use a discrete nonlinear diffusion process for preliminary adaptive smoothing of range images and polygonal surfaces. Filtering by a nonlinear diffusion proposed first by Perona and Malik [13] provides with a very powerful tool for smoothing coupled with feature preserving and enhancing. Since we need a smoothing process preserving and enhancing sharp variation points of the surface normals, we introduce a nonlinear diffusion process acting on the surface normals.

* Address: The University of Aizu, Aizu-Wakamatsu 965-8580 Japan. E-mails: {d8011101,belyaev}@u-aizu.ac.jp

Mathematically sharp variation points of surface normals can be described via extrema of the principal curvatures along their curvature lines. Such extrema have been intensively studied in connection with research on classical differential geometry and singularity theory [10, 14, 1, 8], segmentation of range images [15, 2], image and data analysis [6], face recognition [8], quality control of free-form surfaces [9], analysis of the anatomy of the human skull and brain [12]. See also references therein.

Following [1] we define the *ridges* as the locus of points where the maximal principal curvature attains a positive maximum along its curvature line and the *ravines* as the locus of points where the minimal principal curvature attains a negative minimum along its curvature line. This rigorous mathematical definition is similar to a widely used definition for edges in image processing: the edges consist of pixels where the magnitude of the gradient of the image intensity has a local maximum in the direction of the gradient [5]. Following [2, 4] we use this analogy for detection of ridges and ravines on range data.

Triangle meshes are the most popular way to represent shapes of objects in computer graphics and geometric modeling. Detection of ridges and ravines on a triangular mesh is a challenging task because of mesh irregularity. It turns out that instead of estimating curvature extrema at the mesh vertices it is more effective to define ridges and ravines via dihedral angles between adjacent triangles [4]. Let us call a mesh edge a *sharp edge* if the dihedral angle between two its adjacent triangles is below a certain threshold. We define ridges and ravines on a triangular mesh as the convex and concave sharp edges, respectively, and use a discrete nonlinear diffusion (nonlinear averaging) acting on the mesh normals for enhancing perceptually salient ridges and ravines. However such averaging destroys the integrability of the family of planes orthogonal to the evolving vectors. To enforce the integrability we update the positions of the inner mesh vertices after every step of the averaging procedure in an attempt to fit the surface to the modified family of planes orthogonal the evolving vectors. Then we recompute the mesh normals and start the next pass of the averaging procedure.

Note that changing the surface orientation turns the ridges into the ravines and vise versa. Thus without loss of generality we can consider only the ridges.

2 Nonlinear Diffusion of Normals for Detection of Ridges on Range Images

Consider a range image defined by a height function $I(x, y)$. Let $k_{\max}$ and $k_{\min}$, $k_{\max} > k_{\min}$ be the principal curvatures of the surface $z = I(x, y)$ and $\mathbf{v}_{\max}$ and $\mathbf{v}_{\min}$ be the projections of the associated principal directions $\mathbf{t}_{\max}$ and $\mathbf{t}_{\min}$ onto xy-plane.

To estimate whether $k_{\max}$ takes a maximum along $\mathbf{t}_{\max}$ at a given pixel P we adopt a standard technique used for edge detection [7], see Fig. 1. The principal curvature $k_{\max}$ at Q_1 and Q_2 is estimated by bilinear interpolations between the values of $k_{\max}$ at P_1, P_2 and P_3, P_4, respectively.

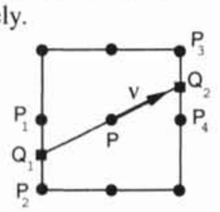

Fig. 1: The principal curvatures at Q_1 and Q_2 are estimated by bilinear interpolations between the curvature values at P_1, P_2 and P_3, P_4, respectively.

We mark pixel P as a ridge pixel if $\mathbf{v} = \mathbf{v}_{\max}$, $k_{\max}(P) > T$, $k_{\max}(P) > k_{\max}(Q_1)$, $k_{\max}(P) > k_{\max}(Q_2)$, where T is a given positive threshold. We use hysteresis thresholding [5] to remove unessential ridge and ravine pixels and keep the ridge and ravine pixels connected as much as possible. We keep a chain of connected ridge pixels with $k_{\max} > T_{\mathrm{lo}}$ if $k_{\max} > T_{\mathrm{hi}}$ for at least one pixel in the chain. Here $T_{\mathrm{hi}} > T_{\mathrm{lo}} > 0$.

This simple approach for detection of the ridges and ravines on range images was proposed in [2] where it was combined with a slight modification of the Saint-Marc and Medioni adaptive smoothing scheme [15] applied to the image derivatives $I_x(x, y)$ and $I_y(x, y)$. However, Saint-Marc and Medioni preliminary filtering is not invariant under rotations and, therefore, is not appropriate for the detection of rotation-invariant range image features.

Our idea is to use a nonlinear diffusion (iterative nonlinear averaging) of the normals of the surface $z = I(x, y)$ instead of smoothing the first-order image derivatives.

The downward unit normal of $z = I(x, y)$ is given by

$$\mathbf{n}(x, y) = [n_1, n_2, n_3] = \frac{[I_x(x, y), I_y(x, y), -1]}{\sqrt{1 + I_x(x, y)^2 + I_y(x, y)^2}}$$

Note that

$$I_x(x, y) = -\frac{n_1(x, y)}{n_3(x, y)}, \quad I_y(x, y) = -\frac{n_2(x, y)}{n_3(x, y)},$$

Let $P(x, y, n)$ and $Q(x, y, n)$ approximate the image derivatives $I_x(x, y)$, $I_y(x, y)$ after n smoothing iterations. The Gaussian and mean curvatures can be approximated by

$$K = \frac{P_x Q_y - P_y Q_x}{(1 + P^2 + Q^2)^2}, \tag{1}$$

$$H = \frac{(1 + P^2)P_x - PQ(P_y + Q_x) + (1 + Q^2)Q_y}{2(1 + P^2 + Q^2)^{3/2}}. \tag{2}$$

For every pixel (x, y) let us define its weight by

$$w(x, y, n) = \exp\{-c\left(2H(x, y, n)^2 - K(x, y, n)\right)\},$$

where c is a positive constant. Routinely we use $c = 0.1$. Consider the following discrete vector-valued diffusion process

$$\mathbf{m}(x, y, n + 1) = \frac{\sum \mathbf{m}(x + i, y + j, n) w(x + i, y + j, n)}{\sum w(x + i, y + j, n)},$$

$$\mathbf{m}(x, y, 0) = \mathbf{n}(x, y),$$

where the summations are taken over a 3 by 3 square neighborhood of (x, y) pixel. The next approximations of the image derivatives are now computed by

$$P(x, y, n + 1) = -m_1(x, y, n + 1)/m_3(x, y, n + 1),$$
$$Q(x, y, n + 1) = -m_2(x, y, n + 1)/m_3(x, y, n + 1),$$

After a number of smoothing iterations the principal curvatures are obtained from the Gaussian and mean curvatures given by (1), (2). The coefficients of the first and second fundamental forms are computed by

$$E = 1 + P^2, \quad L = \frac{P_x}{\sqrt{1 + P^2 + Q^2}},$$
$$F = PQ, \qquad M = \frac{P_y + Q_x}{2\sqrt{1 + P^2 + Q^2}},$$
$$G = 1 + Q^2, \quad N = \frac{Q_y}{\sqrt{1 + P^2 + Q^2}}$$

and computation of the projections $\mathbf{v}_{\max}$ and $\mathbf{v}_{\min}$ of the principal directions onto the image plane is straightforward.

Fig. 2 shows detection of the ridge pixels on a range image of an object with straight and circular edges. Note that the presented method detects well all straight and curvilinear (circular) ridge structures and does not respond to the object self-occlusions. Fig. 3 exposes the ridges and ravines detected on a complex range data.

3 Nonlinear Averaging for Detection of Ridges and Ravines on Triangle Meshes

Consider an oriented triangle mesh. Let T be a mesh triangle and $\mathbf{n}(T)$ be the unit normal of T. Denote by $\mathcal{N}(T)$ the set of all mesh triangles that have a common edge or vertex with T. Consider a nonlinear discrete diffusion process consisting of the following two successive steps applied iteratively.

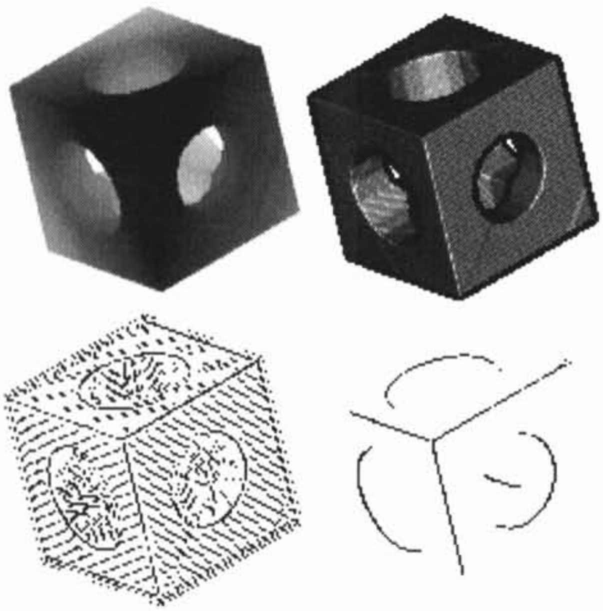

Fig. 2: Top-left: a range image. Top-right: a shaded image of the range data. Bottom-left: the ridge and ravine pixels detected without preliminary smoothing. Bottom-right: discrete nonlinear diffusion of normals was used for preliminary smoothing.

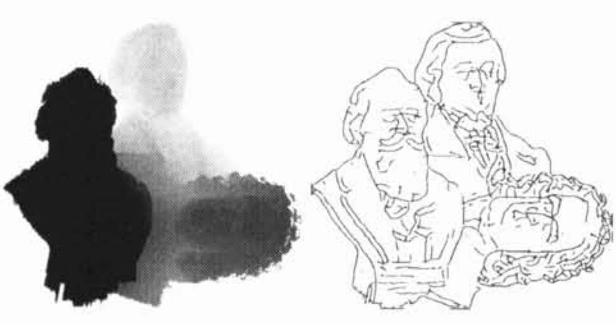

Fig. 3: Ridges and ravines on a complex range image.

Step 1. For all mesh triangles T compute triangle normals $\mathbf{n}(T)$ and perform the following nonlinear averaging procedure

$$\mathbf{m}(T) = \frac{1}{\sum w(T,S)} \sum_{S \in \mathcal{N}(T)} w(T,S)\mathbf{n}(S), \qquad (3)$$

$$\text{with} \quad w(T,S) = \exp\left(-c\,K^2\right), \quad K = \varphi/d. \qquad (4)$$

Here c is a positive constant, $\varphi = \angle\left(\mathbf{m}(T), \mathbf{m}(S)\right)$ is the angle between the vectors $\mathbf{m}(T)$ and $\mathbf{m}(S)$, and d is the distance between the centroids of triangles T and S (see the left and middle images of Fig. 4).

Step 2. For all mesh vertices P perform the vertex updating procedure

$$P_{\text{new}} \longleftarrow P_{\text{old}} + \frac{1}{\sum A(T)} \sum A(T)\mathbf{v}(T),$$

where the sums are taken over all triangles T adjacent to P, $\mathbf{v}(T) = \left[\overrightarrow{PC} \cdot \mathbf{m}(T)\right]\mathbf{m}(T)$ is the projection of the vector $\overrightarrow{PC}$ onto the $\mathbf{m}(T)$ direction, C is the centroid of T (see the right image of Fig. 4), $A(T)$ denotes the area of T.

Parameter c in (4) governs the degree of nonlinearity in the averaging step (n-aver). Fig. 6 demonstrates how this parameter affects the smoothing process.

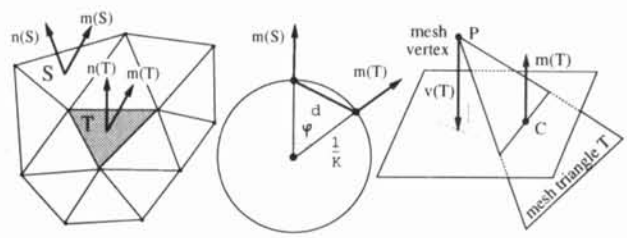

Fig. 4: Left: mesh triangles and their associated vectors. Middle: directional curvature estimation. Right: updating vertex position.

Note that if $\mathbf{m}(T)$ and $\mathbf{m}(S)$ were the usual normals for triangles T and S, then K would approximate the directional curvature at the centroid of T in the direction of the centroid of S. Thus, the first step of the above iterative nonlinear averaging scheme resembles strongly the discrete diffusion process we used in the previous section for range data selective smoothing.

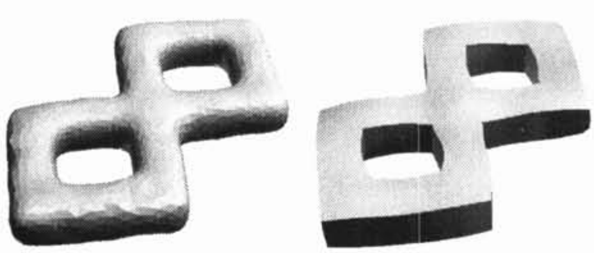

Fig. 5: Left: a polygonal two-holed torus. Right: the ridges are sharpened after adaptive smoothing and provide with a natural decomposition of the two-holed torus.

Our nonlinear averaging scheme sharpens salient ridges and ravines and can be used for natural shape segmentation, see Fig. 5.

For dense triangular meshes approximating shapes of objects accurately, the dihedral angles between triangles shearing a common edge are very close to the straight angle. For the models considered below, the angle threshold for the ridge and ravine edges is equal to 162^o.

Fig. 6 shows detection of ridge and ravine structures on the Venus model smoothed by 100 iterations of the two-step adaptive smoothing method described above. Varying the number of smoothing iterations and the exponent coefficient c in (4) allows us to perform a multiscale shape analysis.

Fig. 7 demonstrates shape smoothing and detection

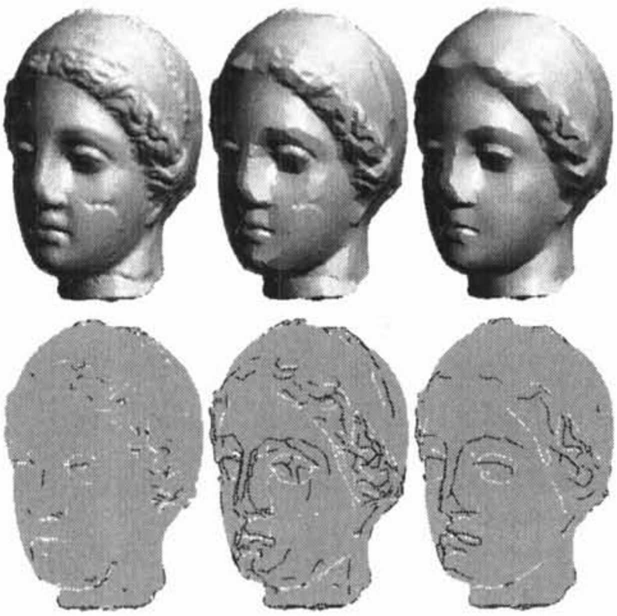

Fig. 6: Top-left: the Venus head model (courtesy Cyberware). Top-middle: the Venus head model smoothed by 100 iterations with $c = 4$. Top-right: the Venus head model smoothed by 100 iterations with $c = 0.7$. Bottom row: ridges (convex sharp edges) and ravine (concave sharp edges) colored in black and white, respectively, are detected on the top row models.

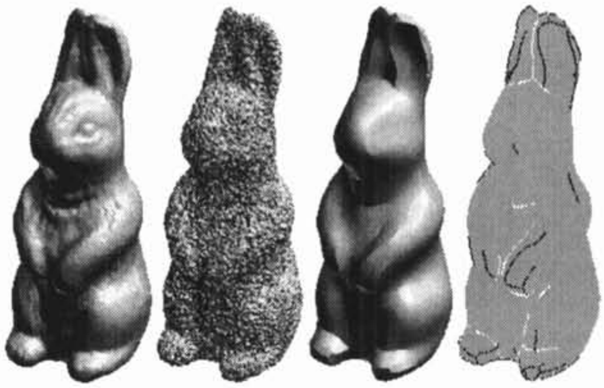

Fig. 7: From left to right: a bunny model (courtesy Cyberware); the model with a random noise added; diffusion of normals with enforcing integrability suppresses the noise and sharpens main ridge and ravine structures; ridges (black) and ravine (white) detected on the smoothed model.

of the ridges and ravines on a noisy polygonal model. Our smoothing scheme suppresses the noise and sharpens main ridge and ravine structures.

4 Conclusion and Future Work

We developed a new surface smoothing method. It preserves sharp variation points of the surface normals and is good for stable extraction of perceptually salient ridges and ravines. The method is based on a nonlinear diffusion process applied to the surface normals. We demonstrated applicability of the proposed smoothing method coupled with ridge and ravine extraction schemes to analysis and segmentation of range images (depth data) and smooth surfaces approximated by triangle meshes.

Many directions for future work remain. In particular, we want to combine the smoothing scheme presented in this paper with our approach for automatic detection of geodesic ridges and ravines [11].

References

[1] A. G. Belyaev, E. V. Anoshkina, and T. L. Kunii. Ridges, ravines, and singularities. In A. T. Fomenko, and T. L. Kunii, *Topological Modeling for Visualization.* Ch. 18, pages 375–383, Springer, 1997.

[2] A. G. Belyaev, I. A. Bogaevski, and T. L. Kunii. Ridges and ravines on a surface and segmentation of range images. In *Vision Geometry VI, Proc. SPIE 3168*, pages 106–114, 1997.

[3] A. G. Belyaev and Yu. Ohtake. An image processing approach to detection of ridges and ravines on polygonal surfaces. In *EUROGRAPHICS 2000, Short Presentations*, pages 19–28, August 2000.

[4] A. G. Belyaev, Yu. Ohtake, and K. Abe. Detection of ridges and ravines on range images and triangular meshes. In *Vision Geometry IX, Proc. SPIE 4117*, July-August 2000.

[5] J. Canny. A computational approach to edge detection. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 8(6):679–698, 1986.

[6] D. Eberly. *Ridges in Image and Data Analysis.* Kluwer, 1996.

[7] O. Faugeras. *Three-Dimensional Computer Vision,* Ch. 4: *Edge Detection.* MIT Press, 1993.

[8] P. L. Hallinan, G. G. Gordon, A. L. Yuille, P. Giblin, and D. Mumford. *Two- and Tree-Dimensional Patterns of the Face.* Ch. 6: *Parabolic Curves and Ridges on Surfaces.* A K Peters, 1999.

[9] M. Hosaka. *Modeling of Curves and Surfaces in CAD/CAM.* Springer, Berlin, 1992.

[10] J. J. Koenderink. *Solid Shape.* MIT Press, 1990.

[11] Yu. Ohtake and A. G. Belyaev. Geodesic ridges and ravines on polygonal surfaces. In *Proceedings of the Third International Conference on Human and Computer*, pages 5–10, September 2000.

[12] X. Pennec, N. Ayache, and J. P. Thirion. Landmark-based registration using features identified through differential geometry. In I. N. Bankman, editor, *Handbook of Medical Imaging.* Academic Press, 2000.

[13] P. Perona and J. Malik. Scale-space and edge detection using anisotropic diffusion. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 12(7):629–638, July 1990.

[14] I. R. Porteous. *Geometric Differentiation for the Intelligence of Curves and Surfaces.* Cambridge University Press, Cambridge, 1994.

[15] P. Saint-Marc and G. Medioni. Adaptive smoothing for feature extraction. In *Proc. of DARPA Image Understanding Workshop*, pages 1100–1113, Cambridge, Massachusetts, April 1988.