# 13—1 Recovering Shape of Unfolded Book Surface from a Scanner Image using Eigenspace Method

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## Abstract

In this study, we address the problem of recovering the cross section shape of an unfolded book from the image of its surface taken by the image scanner. To attack this problem, we propose the practical method using the eigenspaces which are constructed by some shape and intensity profile data of the book surface to reduce the data size. In this method, the book surface shape is recovered from the shading data of the input image by using the linear interpolation considering the error of the linear approximation in the eigenspaces. By using this method, we can recover the book surface shape in practical time, and the geometric and photometric distortions in the image can be removed using the reconstructed shape.

## 1 Introduction

In this paper, we address the problem to recover the shape of an unfolded book surface from an image taken by the image scanner. As for this problem, we have proposed the method using the iterative nonlinear optimization with the precise optical models about the image scanner and the book surface[1]. But this method required so much computation time that it couldn't use for practical. To solve this problem in practical time, here, we propose the another method using the eigenspaces[2].

The eigenspace method is usually used for the 3D object recognition and the pose estimation[2], the visual servoing of the robot[3],[4], and so on. In these problems, the parameters are estimated by the image data projected in the eigenspace. In this paper, we apply these methods to the shape reconstruction problem.

In our method, some shape and shading data of the book surface are preserved in the eigenspaces to reduce the data size, and the book surface shape is recovered from the shading data of the input image by the linear interpolation in the eigenspaces. To estimate the book surface shape accuracy, we introduce the following techniques: (1) the normalization



Figure 1: Structure of image scanner.

of the shape and shading data to make them to the shift invariant profiles, (2) the interpolation including the error terms caused by the linear approximation. By using the reconstructed shape, the geometric and photometric distortions in the scanner image can be removed.

### 2 Image Scanner and Observed Data

Figure 1 shows the configuration of the image scanner and the book surface. The x-axis, the linear CCD sensor, the light source, and the center line separating the book pages are arranged parallel each other. We assume that the book surface is cylindrical, so that the cross section shape is constant along the x-axis. Hence, the reconstructed shape is the 2D cross section shape in the y - z plane.

The shape profile is defined as the sequence of the height between the scanning plane and the book surface, and described as the vector Z:

$$\boldsymbol{Z} = \left[ Z(y_0) \ Z(y_1) \cdots Z(y_{M-1}) \right]^T, \qquad (1)$$

where M is the number of pixels along the y-axis in the scanner image. The shading profile is also defined as the sequence of the intensity at the white background on the book surface, described as the vector P:

$$\boldsymbol{P} = [P(y_0) \ P(y_1) \cdots P(y_{M-1})]^T \,. \tag{2}$$

Figure 2 shows the observed image of the real book surface, and figure 3 shows the shading profile of figure 2.

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Figure 2: Observed image of real book surface.





In this method, we use some shape and shading profiles as the basis profiles. To reduce the type of the basis profiles, the shape and shading profiles are normalized. The shape profiles  $\mathbf{Z}_i$  are moved along the y-axis so that the binding position  $b_i$  which is the position of the center line separating the book pages should be located in the same y position  $b_o = (M - 1)/2$ . The shading profiles  $\mathbf{P}_i$  are also moved along the y-axis so that the phase of the first frequency component  $\phi_{1i}$  should be equal to zero. The new binding positions of the normalized shading profiles  $\hat{b}_i = b_i - \phi_{1i} \cdot M/2\pi$  are used to estimate the binding position of the recovered shape.

#### 3 Shape Reconstruction

In this section we describe the shape reconstruction method from the input shading profile by using the basis profiles. Figure 4 shows the flowchart of the shape reconstruction. First, the eigenspaces of the basis profiles are constructed. Let the set of the normalized basis shading profiles  $\hat{P}_i(1 \leq i \leq N)$ be  $\boldsymbol{\Phi} = [\hat{P}_1 \ \hat{P}_2 \cdots \hat{P}_N] \ (M \times N)$ . The shading eigenspace is computed from the eigenvalues and eigenvectors of the covariance matrix  $\boldsymbol{Q} = \delta \boldsymbol{\Phi} \cdot \delta \boldsymbol{\Phi}^T$ , where  $\delta \boldsymbol{\Phi} = [\hat{P}_1 - \boldsymbol{m}_P \ \hat{P}_2 - \boldsymbol{m}_P \cdots \hat{P}_N - \boldsymbol{m}_P]$ , and  $\boldsymbol{m}_P$  is the average vector of  $\hat{P}_i$ . Let  $\boldsymbol{E}_P$  be the set of K eigenvectors correspond to the largest K eigenvalues  $(K \ll M)$ . By computing

$$\boldsymbol{G}_{\mathrm{P}} = \boldsymbol{E}_{\mathrm{P}}^{T} \cdot \delta \boldsymbol{\Phi} = [\boldsymbol{g}_{\mathrm{P1}} \ \boldsymbol{g}_{\mathrm{P2}} \cdots \boldsymbol{g}_{\mathrm{PN}}] \quad (K \times N), \quad (3)$$



Figure 4: Flowchart of shape reconstruction.

 $\hat{\boldsymbol{P}}_i$  is projected to the coordinate  $\boldsymbol{g}_{\mathrm{P}i}$  in the shading eigenspace.

Similarly, the shape eigenspace is computed from the set of the normalized basis shape profiles,  $\Psi = [\hat{Z}_1 \ \hat{Z}_2 \cdots \hat{Z}_N]$ , correspond to  $\Phi$ . Here, we denote the average vector as  $m_Z$ , the set of K' eigenvectors as  $E_Z$ , and the set of projected points of the shape profiles as  $G_Z = [g_{Z1} \ g_{Z2} \cdots g_{ZN}]$ .

The shape reconstruction process is as follows. The shading profile  $\boldsymbol{P}^*$  of the input scanner image is normalized to  $\hat{\boldsymbol{P}}^*$  and projected to  $\boldsymbol{g}_{\mathrm{P}}^*$  in the shading eigenspace by

$$\boldsymbol{g}_{\mathrm{P}}^{*} = \boldsymbol{E}_{\mathrm{P}}^{T} \cdot (\hat{\boldsymbol{P}}^{*} - \boldsymbol{m}_{\mathrm{P}}). \tag{4}$$

Let the *L* nearest neighbor points of  $g_P^*$  among the elements of  $G_P$  be  $G_{Ps} = [g_{Ps1} \cdots g_{PsL}]$ , and the subset of  $G_Z$  correspond to  $G_{Ps}$  be  $G_{Zs} = [g_{Zs1} \cdots g_{ZsL}]$ . By using a coefficient vector  $A = [a_1 \cdots a_L]$  which is obtained by  $g_P^*$  and  $G_{Ps}$  as below, the point  $g_Z^*$  in the shape eigenspace correspond to  $g_P^*$  is described as:

$$\boldsymbol{g}_{\mathrm{Z}}^{*} = \sum_{i=1}^{L} a_{i} \cdot (\boldsymbol{g}_{\mathrm{Zs}i} - \boldsymbol{m}_{\mathrm{gZs}}), \qquad (5)$$

where  $m_{gZs}$  is the average vector of  $g_{Zsi}$ . Then, the reconstructed shape profile  $\hat{Z}^*$  is obtained by the inverse projection of

$$\bar{\boldsymbol{Z}}^{*} = \boldsymbol{E}_{\mathrm{Z}} \cdot \boldsymbol{g}_{\mathrm{Z}}^{*} + \boldsymbol{m}_{\mathrm{Z}}.$$
 (6)

The coefficient vector A is estimated as follows. First, we assume the non-linear relation f between  $g_{\mathrm{P}i}$  and  $g_{\mathrm{Z}i}$  as  $g_{\mathrm{P}i} = f(g_{\mathrm{Z}i})$ , and approximate the linear projection including the error term in the local area around  $m_{\mathrm{gZs}}$ :

$$\boldsymbol{g}_{\mathrm{Ps}i} - f(\boldsymbol{m}_{\mathrm{gZs}}) = F(\boldsymbol{m}_{\mathrm{gZs}})(\boldsymbol{g}_{\mathrm{Zs}i} - \boldsymbol{m}_{\mathrm{gZs}}) + \boldsymbol{v}_{\mathrm{s}i}, \ (7)$$

where  $F(\boldsymbol{m}_{gZs}) = [\partial f/\partial \boldsymbol{g}_{Zi}]\boldsymbol{g}_{Zi} = \boldsymbol{m}_{gZs}$ , and  $\boldsymbol{v}_{si}$  is the sum of the error in the approximation of the linear projection and in the observed data. Next, the following linear interpolation about  $\boldsymbol{g}_{P}^{*}$  and  $\boldsymbol{g}_{Psi}$  is led



Figure 5: Book surface model.

from equation (5) and (7):

$$(\boldsymbol{g}_{\mathrm{P}}^{*}-\boldsymbol{m}_{\mathrm{gPs}})-\boldsymbol{v}^{*}=\sum_{i=1}^{L}a_{i}\cdot((\boldsymbol{g}_{\mathrm{Ps}i}-\boldsymbol{m}_{\mathrm{gPs}})-\boldsymbol{v}_{\mathrm{s}i}),\ (8)$$

where  $m_{\rm gPs}$  is the average vector of  $g_{\rm Psi}$ .

From equation (8), A is obtained by minimizing the error  $v^*$ :

$$\boldsymbol{A} = \left[ \Delta \boldsymbol{G}_{\mathrm{Ps}}^{T} \cdot \Delta \boldsymbol{G}_{\mathrm{Ps}} + \sigma^{2} \cdot \boldsymbol{K} \cdot \boldsymbol{I} \right]^{-1} \\ \cdot \Delta \boldsymbol{G}_{\mathrm{Ps}}^{T} \cdot (\boldsymbol{g}_{\mathrm{P}}^{*} - \boldsymbol{m}_{\mathrm{gPs}}), \qquad (9)$$

where  $\Delta \boldsymbol{G}_{\mathrm{Ps}} = [\boldsymbol{g}_{\mathrm{Ps1}} - \boldsymbol{m}_{\mathrm{gPs}} \cdots \boldsymbol{g}_{\mathrm{PsL}} - \boldsymbol{m}_{\mathrm{gPs}}]$ , and  $\sigma^2 \cdot \boldsymbol{I}$  is the covariance matrix of the errors. Let  $\nu$  be the signal-to-noise ratio of the elements in  $\Delta \boldsymbol{G}_{\mathrm{Ps}}$ ,  $\sigma^2$  is obtained by

$$\sigma^2 = \nu^2 \cdot \frac{\operatorname{tr} \left[ \Delta \boldsymbol{G}_{\mathrm{Ps}}^T \cdot \Delta \boldsymbol{G}_{\mathrm{Ps}} \right]}{KL}.$$
 (10)

The binding position of  $\hat{Z}^*$  is  $b_o$ , so the actual binding position  $b^*$  is estimated by the linear interpolation using A as follows:

$$b^* = \begin{bmatrix} (\hat{b}_{s1} - m_b) & \cdots & (\hat{b}_{sL} - m_b) \end{bmatrix} \cdot \boldsymbol{A} + m_b + \phi_1^* \cdot M/2\pi$$
(11)

where  $b_{si}$  is the binding position of  $\hat{P}_i$  correspond to  $g_{Psi}$ ,  $m_b$  is the average of  $\hat{b}_{si}$ , and  $\phi_1^*$  is the phase of the first frequency component of  $P^*$ .

# 4 Experiments

In this section, we show the experimental results of the shape reconstruction. The basis profiles of shape and shading are acquired by using the book surface model (figure 5). The shape profiles are measured by the arm-typed 3D digitizer, and the shading profiles are extracted from the scanner images of the model's surface. We use 64 pairs of the shape and shading profiles as the bases. These profiles are normalized and the eigenspaces are computed. Figure 6 and 7 show the eigenvectors of the shading and shape profiles. Figure 7 shows the feature of the shape variation and figure 6 is the intensity variation correspond to each shape eigenvector.

To confirm the accuracy of the proposed method, we use the 150 scanner images and its shapes of the book surface model which are different from the bases, and evaluate the error of the reconstructed shape. Figure 8 shows the average of the errors of the 150 reconstructed shapes varying the SN ratio



Figure 6: Eigenvectors of shading profiles (No.1~4).



Figure 7: Eigenvectors of shape profiles (No.1~4).



Figure 8: Reconstructed shape error varying with signal-to-noise ratio.



Figure 9: Reconstructed shape error varying with eigenspace dimension.



Figure 10: Histogram of shape error.



Figure 11: Estimated shapes.

 $(\nu)$  and L (where K = 10). Figure 9 also shows the average of the errors varying K, K' and L (where  $\nu = 10\%$ ). From these results, by considering the error of the linear approximation, the shape profile can be recovered accuracy, and  $L = 13, K, K' = 20, \nu = 10\%$  are the best for this experiment.

Figure 10 shows the histogram of the reconstructed shape errors in case of (a) the proposed method, (b) not normalizing the profiles, and (c) the nearest neighborhood method. From this result, the proposed method can reconstruct the shapes more accurately than other methods. Figure 11 shows the estimated shape profiles and the real shape profile. By using the proposed method, the estimated shape is almost matched to the real shape.

Figure 12 shows the reconstructed shape profile when figure 2 is used as the input image. For this result,  $K, K' = 20, L = 13, \nu = 10\%$ . Figure 13 is the restored image using the estimated shape. Because there is no texture around the binding position in figure 2, the shape is estimated a little lower than the actual shape, but the readability of the book surface is improved.

The computation time for the shape reconstruction and the image restoration is about 2 seconds. It become drastically faster than the method we proposed before[1].







Figure 13: Restored image of real book surface.

#### 5 Conclusion

In this paper, we discuss the problem to restore the book surface shape from the scanner image. In the proposed method, the basis shape and shading profiles are restored in the eigenspaces to reduce the data size, and the linear interpolation in the eigenspaces is used to recover the shape profile from the input shading profile. By using the proposed method, we can recover the book surface shape and its image in practical time. To improve this method, we must consider the difference of the reflectance properties of the various papers.

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