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# Curve Extraction Using Genetic Algorithm Based on Closeness and Continuity in Perceptive Grouping Factors

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### Abstract

A method to extract a visual continuous curve using a genetic algorithm is proposed in an image including a discontinuous curve and noisy points. The fitness of an individual in the population is defined by the global shape of a curve that is formed by connecting all points extracted by the individual based on proximity and continuity of points in perceptive grouping factors. A final shape of a curve that is formed by connecting points included in the individual with the maximum fitness is extracted after evolution of a genetic algorithm. Experimental results show that the proposed method extracted visual perceptive curves in noisy images.

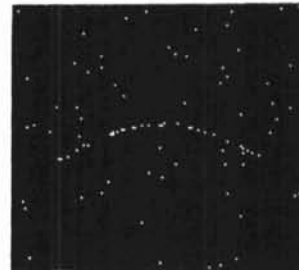


Fig.1 A sample objective image

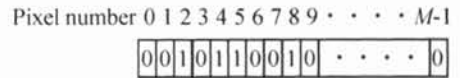


Fig.2 A chromosome in an individual

### 1. Introduction

Extracting a global contour of a figure is an important processing in image recognition. A contour often becomes discontinuous and many noisy pixels occurs in a low contrast image. However, the human visual ability can perceive a subjective curve by connecting independent pixels in an image including many pixels[1] ~ [3]. This visual function is materialized by perceptive grouping factors in Gestalt psychology[4]. The process to select only pixels to compose a subjective curve and the process to connect these pixels to the correct order are required to extract a visual continuous curve in an image including many noisy pixels[5]. Namely, a method to search a solution as an appropriate visual curve is needed. To search a visual curve, to recognize a global curve by grouping independent pixels is effective using perceptive grouping factors. The method to connect elements of a contour was proposed by regulating perceptive grouping factors[6]. However, this method is restricted that the interval of each element is approximately constant.

In this paper, a method to extract a visual continuous

curve in an image including noisy pixels using a genetic algorithm(GA) based on perceptive grouping factors. GA is a method to search a solution for an engineering theme by aiming evolutionary rules of life[7]. However the methods to extract closed loops using GA based on perceptive grouping factors[8]~ [10], this paper describes a general method to extract arbitrary visual curves.

### 2. Curve extraction using GA

The proposed method to extract a curve is described below. The purpose of processing is to extract a continuous visual curve in an image including many white pixels as shown in Fig.1. The pixel number is assigned to all white pixels in the image sequentially from the upper left corner.

#### 2.1 Definition of chromosome

Pixels for candidates to compose a curve are represented by an arrayed bits as a chromosome of an individual in the population. The proper number of white pixels  $N$  to compose a curve are desirable to change dynamically in an evolutionary process in GA. Thereupon, the chromosome of an individual is represented by the one-dimensional arrayed bits with

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Fig.3 Closeness factor



Fig.4 Continuous factor

the length  $M$  that is the number of all white pixels in the objective image, as shown in Fig.2. Each bit in the arrayed bits corresponds with each white pixel in the objective image. Therefore, the white pixels in the image that correspond to the bits are selected as pixels for candidates to compose a curve.

**2.2 Perceptive grouping factors**

A collection of individual elements is perceived as a global figure by perceptive grouping factors. The closeness, the continuity, the similarity, the symmetry etc. among elements are cited as perceptive grouping factors[4]. In the proposed method, the global form of a curve is evaluated by the closeness and the continuity of pixels for candidates.

**2.2.1 Closeness**

In the case that some pixels are allocated to a near position mutually as shown in Fig.3(a), these pixels are perceived as a group by the closeness factor. On the other hand, in the case that some pixels are allocated to a long distance mutually as shown in Fig.3(b), each pixel is perceived as independent element. As the averaged distance between neighbor pixels to compose a curve is smaller, the curve is evaluated to have a visual appropriate form by the closeness factor.

**2.2.2 Continuity**

In the case that some pixels are allocated on a smooth curve as shown in Fig.4(a), these pixels are perceived as a group by the continuity factor compared with the pixels allocated irregularly as shown in Fig.4(b). To evaluate a continuity of a curve, each inner angle at a pixel between neighbor pixels is measured. As the averaged inner angle between neighbor pixels to compose a curve is larger, the curve is evaluated to have a visual smooth form by the continuity factor.

**2.3 Extraction of candidate for edge pixel**

The following explanation describes the process to extract a candidate for an edge pixel in pixels that compose a curve. As shown in Fig.5, a pixel  $P_i$  that is

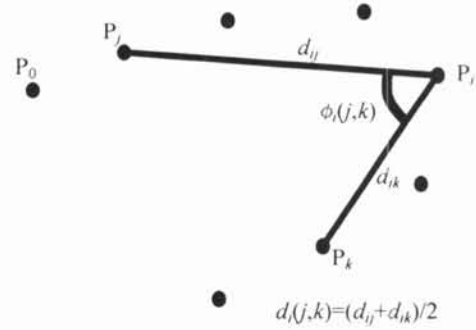


Fig.5 Distances and inner angle between points in processing for extracting an edge pixel

allocated on the way of a curve has the condition that the inner angle between other two pixels is large and the averaged distance to these two pixels is short. On the other hand, the edge pixel  $P_0$  of the curve does not have such a condition. Namely, the edge pixel tends to be perceived as the isolated pixel in comparison with pixels on the way of the curve by the closeness factor and the continuity factor. The averaged distance  $d(j,k)$  from the pixel  $P_i$  to other two pixels  $P_j$  and  $P_k$  and the inner angle  $\phi(j,k)$  between the pixel  $P_j$  and the pixel  $P_k$  at the pixel  $P_i$  are measured as shown in Fig.5. Next, the value  $E_i(j,k)$  to evaluate the degree that the pixel  $P_i$  is allocated on the way of the curve is calculated by the next expression in the relative location between the pixel  $P_j$  and the pixel  $P_k$ .

$$E_i(j,k) = 1 / \{d(j,k) + \eta (\pi - \phi(j,k))\} \quad (1)$$

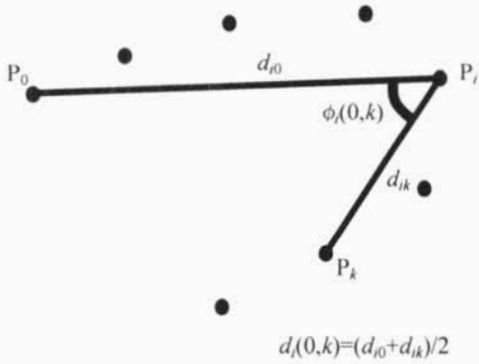
Here,  $\eta$  is a positive constant. Next, the maximum value  $E_{i,max}$  in all  $E_i(j,k)$  that are obtained by all combinations of  $j$  and  $k$  is gained by the next expression.

$$E_{i,max} = \max E_i(j,k) \quad (j=0,1,\dots,N-1, k=0,1,\dots,N-1) \quad (2)$$

If the pixel  $P_i$  is on the way of the curve,  $E_{i,max}$  becomes a large value because the condition occurs that  $\phi(j,k)$  becomes a large value and  $d(j,k)$  becomes a small value in relative locations of all pixels  $P_j$  and  $P_k$ . On the other hand, if the pixel  $P_i$  is the edge pixel of the curve,  $E_{i,max}$  becomes a smaller value relatively. The pixel that has the minimum  $E_{i,max}$  value in all pixels  $P_i (i=0,1,2,\dots,N-1)$  is regarded as the candidate for the edge pixel  $P_0$ .

**2.4 Definition of fitness**

In the case of the number of bits 1 included in a chromosome of an individual is  $N$ , the fitness of the individual to the environment is determined by the global form of the curve composed by a set of  $N$



**Fig.6** Distances and inner angle between points in processing for connecting points

pixels that correspond to bits 1. The global form of the curve is evaluated by the closeness and the continuity in perceptive grouping factors[8]~[10].

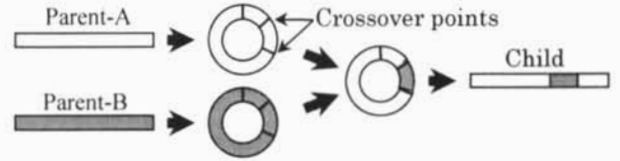
Some pixels are allocated as the pixel  $P_0$  is the edge pixel as shown in Fig.6. The edge pixel  $P_0$  is connected with other arbitrary pixel  $P_i$ , and the pixel  $P_i$  is connected with the arbitrary pixel  $P_k$  except the edge pixel  $P_0$ . The value  $C_i(0,k)$  to evaluate the degree that the pixel  $P_i$  is on the way of the curve is calculated by the next expression in the relative location between the pixel  $P_0$  and the pixel  $P_k$ .

$$C_i(0,k) = 1 / \{d_i(0,k) + \varepsilon (\pi - \phi_i(0,k))\} \quad (3)$$

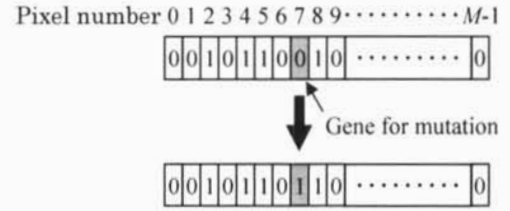
Here,  $d_i(0,k)$  means the averaged distance from the pixel  $P_i$  to the pixel  $P_0$  and to the pixel  $P_k$ , and  $\phi_i(0,k)$  means the inner angle between the two pixels  $P_0$  and  $P_k$  at the pixel  $P_i$ . Also,  $\varepsilon$  is a positive constant. The values  $C_i(0,k)$  for all combinations of the pixel  $P_i$  and the pixel  $P_k$  are calculated. In the case of the value of  $C_i(0,k)$  is the maximum, the pixel  $P_1(=P_i)$  that is connected from the pixel  $P_0$  and the pixel  $P_2(=P_k)$  that is connected from the pixel  $P_1$  are determined. Next, the pixel  $P_2$  is connected with the arbitrary pixel  $P_k$  ( $k \neq 0,1$ ) and the value of  $C_2(1,k)$  at the pixel  $P_2$  is calculated. In the case of the value of  $C_2(1,k)$  is the maximum, the pixel  $P_3(=P_k)$  that is connected from the pixel  $P_2$  is determined.

By the same method mentioned above, the curve is composed by connecting the pixels to the number of  $N$ . The averaged distance  $D$  and the averaged inner angle  $\Phi$  between neighbor two pixels at all pixels that compose the curve are calculated by the next expressions.

$$D = \sum_{i=0}^{N-1} d_i / N \quad (4)$$



**Fig.7** Two-point crossover



**Fig.8** Mutation

$$\Phi = \sum_{i=0}^{N-1} \phi_i / N \quad (5)$$

Here,  $d_i$  means the distance between the pixel  $P_i$  and the pixel  $P_{i+1}$ , and  $\phi_i$  means the inner angle at the pixel  $P_i$ . As the value of  $D$  is smaller, the curve is evaluated to have a visual appropriate form by the closeness factor. Also, as  $\Phi$  is larger, the curve is evaluated to have a visual smooth form by the continuity factor.

The fitness  $F(n)$  of an individual  $n$  is defined by the next expression.

$$F(n) = 1 / \{D(n) + \varepsilon (\pi - \Phi(n))\} \quad (6)$$

Here,  $D(n)$  means the averaged distance between all neighbor pixels and  $\Phi(n)$  means the averaged inner angle at all pixels on the curve that is represented by the chromosome in the individual  $n$ .

### 2.5 Evolutionary operations

The number of individuals in the population is made  $G$ . The individuals to the number of  $G \cdot S/100$  are selected in the population and survived to the next generation with the survived rate  $S\%$ . In this case, fitness of all individuals are calculated by the expression(6) and only a part of individuals with the higher  $S\%$  fitness are selected. On the other hand, the individuals to the number of  $G(100-S)/100$  with the lower  $100-S\%$  fitness are extinguished. Next, two parents are selected in the survived individuals to the next generation, and a child individual is generated by crossover. The children to the number of  $G(100-S)/100$  that is the same as the number of extinguished individuals are generated. Accordingly, the number of individuals in the population  $G$  is kept a constant in all generations.

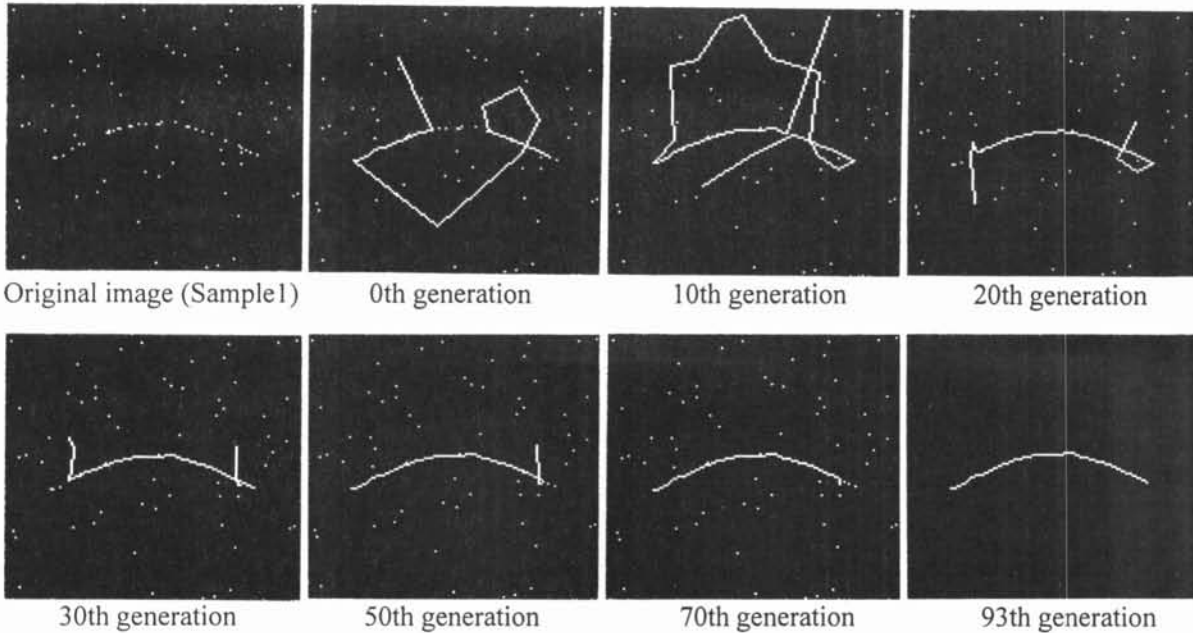


Fig.9 A process of extracting a curve

### 2.6 Crossover

A parent-A and a parent-B are selected at random in the individuals that are survived to the next generation and these parents are crossed over using the two-point crossover method as shown in Fig.7. The chromosomes of the parent-A and the parent-B are separated and connected at the random two positions, and the child individual that has a new chromosome is generated.

### 2.7 Mutation

As shown in Fig.8, a gene is selected at random with the mutation rate  $M\%$  in chromosomes of all individuals, and the bit of the gene is reversed.

### 2.8 Conditions to complete GA

GA is applied by the method and the rule mentioned above. An array of random bits is assigned to the chromosome of each individual in the first generation. When the maximum fitness in all individuals keeps a constant in ten generations, the chromosome of the individual with the maximum fitness expresses the final solution. Namely, the pixels corresponded to bits 1 in the chromosome are selected, and the curve that is connected with these pixels becomes the continuous visual curve as the final solution.

## 3. Experiments and results

### 3.1 Experimental conditions

The size of experimental images is  $120 \times 128$  pixels. Only 0.3% of pixels of backgrounds are random noisy pixels. Only 75% of pixels on continuous curves are erased at random. The parameters for the proposed

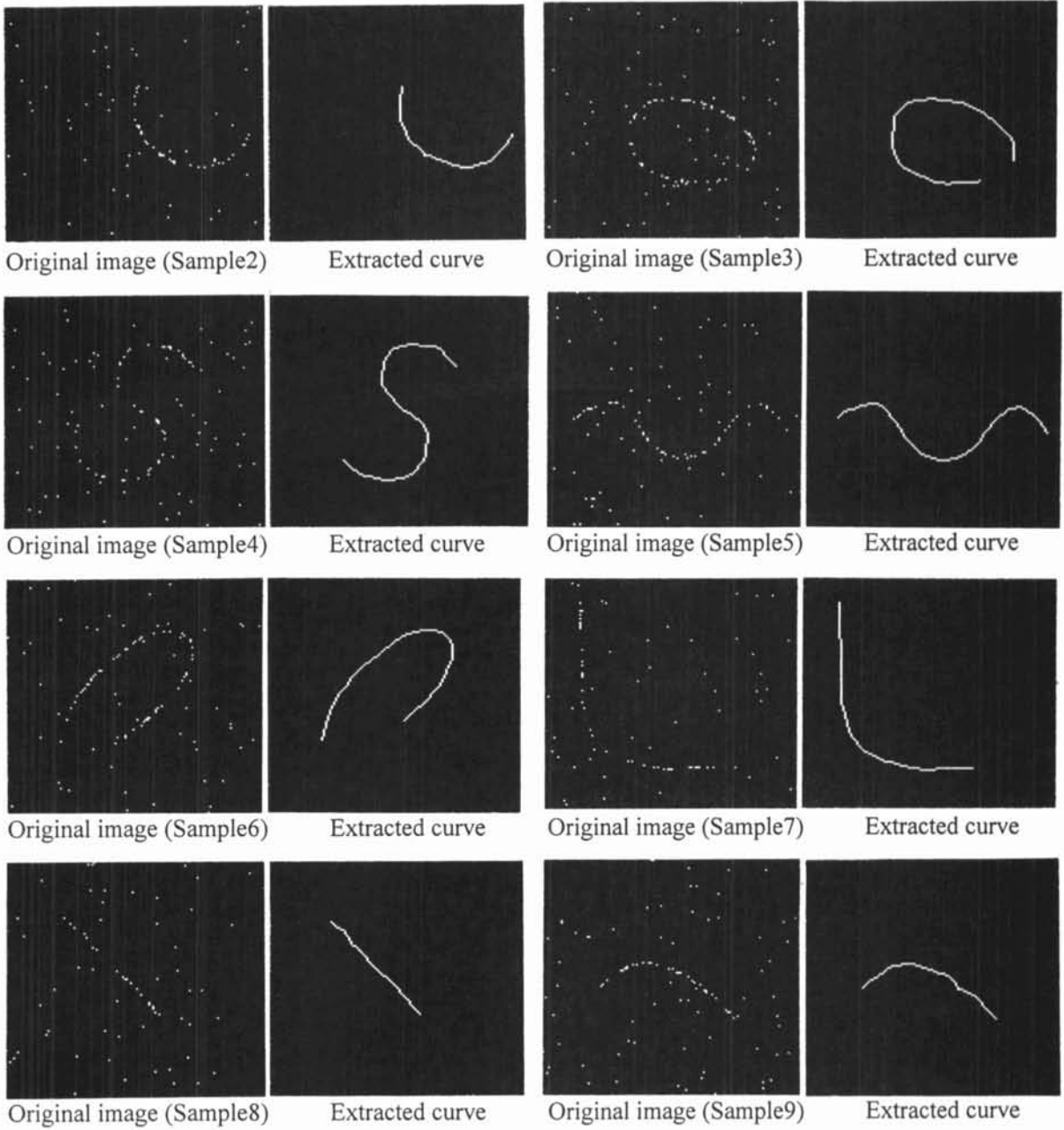
method were set as follows;  $\eta = 10$  as the coefficient in the expression (1),  $\varepsilon = 3$  as the coefficient in the expression (3),  $G = 500$  as the number of individuals in the population,  $S = 50\%$  as the survived rate to the next generation,  $M = 0.1\%$  as the mutation rate and the ratio of bit 1 and bit 0 in the initial chromosomes was 1 to 8.

### 3.2 Processes and results of curves extraction

Fig.9 shows a process to extract a curve in the experimental image(Sample1). A visual appropriate curve had been extracted as generations advanced. Fig.10 also shows the extracted curves in the experimental images(Sample2~9). The curves were extracted that were similar to perceived curves by the visual sense. Fig.11 shows the relations between generations and the maximum fitness in the above curves extraction processes in the experimental images. The final solutions were obtained after 60 to 100 generations in all experimental images.

### 3.3 Change of the number of extracted pixels

Fig.12 shows relations between generations and the numbers of extracted pixels in above curve extraction processes. The number of extracted pixels in each experimental image was converged as generations advanced. Namely, the number of extracted pixels had been changed dynamically in the evolutionary process. Table1 shows the comparison between the numbers of extracted pixels by the visual sense  $n_v$  and the numbers of extracted pixels by the proposed method  $n_p$  to compose the continuous curves. The similar degrees  $C_p$  between  $n_v$  and  $n_p$  were calculated by the next expression.



**Fig.10** Extracted curves by the proposed method

$$C_p = 1 - |n_v - n_p| / n_v \quad (7)$$

The visual appropriate numbers of pixels were extracted in all experimental images.

**3.4 Comparison with random search**

Next, curves were extracted by the random search to compare the proposed method. The number of individuals was set to  $G=500$ . Fig.13 shows relations between generations and the maximum fitness in the random search. Practical solutions were not obtained after searching with 100 generations. These results mean that searching solutions by GA in the proposed

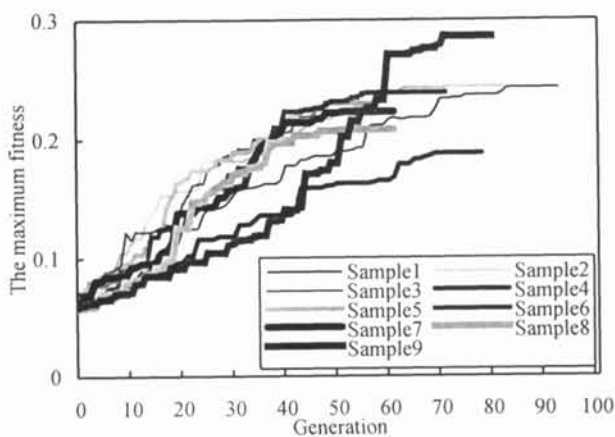
method was working effectively.

**3.5 Processing time**

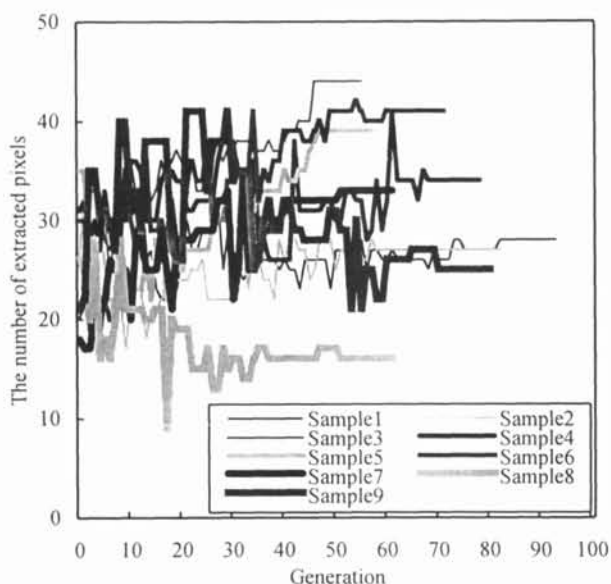
The proposed method spent 3.03 seconds per one generation in evolution by Pentium II (333MHz) processor. Accordingly, the total processing time to extract final curves was from 3 to 4.5 minutes.

**4. Conclusions**

The method to extract a visual continuous curve in an image including a discontinuous curve and noisy pixels using GA that evaluates the fitness of an



**Fig.11** Relation between generation and the maximum fitness



**Fig.12** Relation between generation and the number of extracted pixels

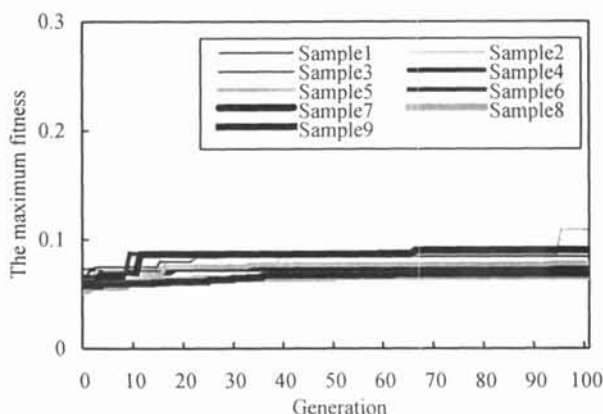
individual based on the closeness and the continuity in perceptive grouping factors is proposed. The continuous curves that were similar to the curves perceived by the visual sense were extracted in the experimental images. It was a difficult theme to extract a global continuous curve using only local information about relative locations of many pixels conventionally. However, global practical solutions were obtained in the enormous combinations for selecting pixels to compose visual curves using GA.

## References

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**Table1** Comparison of the number of pixels extracted by the visual perception and by the proposed method

Experimental images	The number of extracted pixels by the visual sense: $n_v$	The number of extracted pixels by the proposed method: $n_p$	Similar degree between $n_v$ and $n_p$ : $C_p$
Sample1	29	28	0.97
Sample2	26	27	0.96
Sample3	44	44	1.00
Sample4	37	34	0.92
Sample5	38	39	0.97
Sample6	45	41	0.91
Sample7	34	33	0.97
Sample8	17	16	0.94
Sample9	26	24	0.92



**Fig.13** Relation between generation and the maximum fitness by random search

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