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A Relaxed Fuzzy ISODATA Algorithm

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Abstract

In this research, the traditional fuzzy ISODATA (FI) algorithm is integrated with probabilistic relaxation labeling (PRL) algorithm to form a new clustering algorithm called relaxed fuzzy ISODATA (RFI). During the clustering process, both the fuzzy membership function values and local contextual information are employed for grouping data into clusters. The RFI algorithm is considered for clustering noisy data. The experimental results shows the effectiveness of this algorithm.

1. Introduction

Clustering methods have been used extensively in computer vision. In [1], adaptive fuzzy clustering algorithm is employed to detect lines in digital images. Fuzzy c-shell (FCS) algorithm is used for detection of curved boundaries in [2]. In [3], several fuzzy shell clustering algorithms are employed for detecting linear and quadric shell clusters and the issue of cluster validity is also being investigated.

Probabilistic relaxation labeling (PRL) was originally proposed in [4] for reducing the ambiguity in object labeling and has been enhanced in various aspects by many scientists. In [5] the updating rules and compatibility coefficients are derived from basic probability theory. The relaxation process is treated as an optimization problem in [6]. In [7], the stopping criteria of the relaxation process is being investigated. In [8] a new compatibility and support function based on compound statistical decision rules are developed. An excellent survey on PRL algorithm can be found in [9]. Applications of PRL includes curve enhancement [10], edge detection [11], edge matching [12], and so on.

In many clustering based computer vision applications (e.g. segmentation), clustering process usually suffers from attenuation caused by noise. Our objective is to develop a new clustering algorithm which can reduce the attenuation by the utilization of local contextual information. Hence, in this research, PRL technique is incorporated into the traditional fuzzy ISODATA algorithm to form a new kind of clustering algorithm. We name it as relaxed fuzzy ISODATA algorithm (RFI). In section 2, the traditional fuzzy ISODATA algorithm is described in detail. In section 3, a review of the PRL algorithm is presented. Subsequently, the RFI algorithm is described in section 4. Finally, experiments for demonstrating the effectiveness of the RFI algorithm are presented in section 5.

2. Fuzzy ISODATA Algorithm

The traditional fuzzy ISODATA algorithm is summarized as follows:

1. Set up a fuzzy partition u(·) of k non-empty membership functions, $u_i(\cdot)\neq 0$, $1 \le i \le k$ and $2 \le k \le n$ where n is the number of elements in the data set X.

2. The k weighted means are calculated by using the following formula:

$$v_i = \sum_{x\in X} (u_i(x))^2 \times \frac{x}{\sum_{x\in X}(u_i(x))^2}, \quad 1\le i \le k \text{ and } x\in X \qquad (2.1)$$

3. A new partition, $\hat{u}(\cdot)$, is constructed as follows:

Let $I(x) = \{ 1 \le i \le k \mid v_i = x \}$, if $I(x) = \emptyset$, put

$$\hat{u}_j(x) = \sum_{i=1}^{k} \frac{1/\|x-v_j\|^2}{1/\|x-v_i\|^2} \qquad (2.2)$$

else let $\hat{i}$ be the least integer in I(x) and put

$$\hat{u}_i(x) = \begin{cases} 1 & \text{if } i=\hat{i} \\ 0 & \text{if } i\neq\hat{i} \end{cases} \qquad (2.3)$$

for $1 \le i \le k$;

4. If the difference between u(·) and $\hat{u}(\cdot)$ is less than a specific threshold, T, then stop; otherwise, set u(·) to $\hat{u}(\cdot)$ and then go to step 2.

Readers can find more details in [13].

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3. Probabilistic Relaxation Labeling

In this section, the non-linear probabilistic relaxation labeling algorithm introduced in [4] is presented. Consider a set of objects a_i, i = 1, 2,..., N. Object a_i can be labeled as one of the m possible classes from the set $\Lambda = \{ \lambda_k | k = 1,..,m \}$. The objective of the PRL is to assign label θ_i to objects a_i, i = 1,...,N. The compatibility coefficient $c(\theta_i=\lambda_l, \theta_j=\lambda_k)$ represents the contextual information conveyed by label λ_k at object a_j about label λ_l at object a_i. Let the current (qth) estimate of the probability that node a_j has label λ_k be $P^q(\theta_j=\lambda_k)$. Then the support for label λ_l at a_i given by object a_j can be defined as

$$s_j^q(\theta_i=\lambda_l)=\sum_{k=1}^{m} c(\theta_i=\lambda_l,\theta_j=\lambda_k)P^q(\theta_j=\lambda_k) \quad (3.1)$$

and the total support is defined as

$$S^q(\theta_i=\lambda_l)=\sum_{j=1}^{N} W_{ij}\sum_{k=1}^{m} c(\theta_i=\lambda_l,\theta_j=\lambda_k)P^q(\theta_j=\lambda_k) \quad (3.2)$$

where W_{ij} are weights satisfying $\sum_{j=1}^{N} W_{ij}=1$ (3.3)

Given the current value of probability $P^q(\theta_i=\lambda_l)$, the (q+1)th estimate $P^{q+1}(\theta_i=\lambda_l)$ is calculated by

$$P^{q+1}(\theta_i=\lambda_l)=p^q(\theta_i=\lambda_l)\times \frac{S^q(\theta_i=\lambda_l)}{\sum_{k=1}^{m} P^q(\theta_i=\lambda_k)S^q(\theta_i=\lambda_k)} \quad (3.4)$$

4. Relaxed Fuzzy ISODATA Algorithm

Since grouping a data item to a cluster is equivalent to assigning a label to it, the number of labels can be taken as the number of cluster centers. Further, we define $P(\theta_i=\lambda_k)=\frac{1}{\|x_i-v_k\|^2}$. As individual data item is not necessary identified, x denotes x_i in the following. The relaxed fuzzy ISODATA (RFI) algorithm is presented as follows:

1. Set up a fuzzy partition u(·) of k non-empty membership functions, $u_i(\cdot)\neq 0$, $1\le i\le k$ and $2\le k\le n$ where n is the number of elements in the data set X.

2. The k weighted means are calculated by using the following formula:

$$v_i=\sum_{x\in X}(u_i(x))^2\times\frac{x}{\sum_{x\in X}(u_i(x))^2}, \quad 1\le i\le k$$

and $x\in X$ (4.1)

3. Compute

$$R(\theta_i=\lambda_l)=\frac{S^q(\theta_i=\lambda_l)}{\sum_{k=1}^{m} P^q(\theta_i=\lambda_k)S^q(\theta_i=\lambda_k)} \quad \forall i \quad (4.2)$$

using the formulas stated in the previous section. For convenience, we simply denote $R(\theta_i=\lambda_l)$ as $R(x, \lambda_l)$ for an arbitrary data item x.

4. A new partition, $\hat{u}(\cdot)$, is constructed as follows:

Let $I(x) = \{ 1\le i\le k \mid v_i = x \}$, if $I(x) = \varnothing$, let

$$u'_j(x)=\alpha\sum_{i=1}^{k}\frac{1/\|x-v_j\|^2}{1/\|x-v_i\|^2}+\beta R(x,\lambda_j)\text{ ,where}$$

$0\le\alpha\le 1$, $0\le\beta\le 1$ and $\alpha+\beta=1$ (4.3)

put

$$\hat{u}_j(x)=\frac{u'_j(x)}{\sum_{i=1}^{k}u'_i(x)} \quad (4.4)$$

else let $\hat{i}$ be the least integer in I(x) and put

$$\hat{u}_i(x)=\begin{cases}1 & \text{if } i=\hat{i}\\ 0 & \text{if } i\neq\hat{i}\end{cases} \quad (4.5)$$

for $1\le i\le k$;

5. If the difference between u(·) and $\hat{u}(\cdot)$ is less than a specific threshold, T, then stop; otherwise, set u(·) to $\hat{u}(\cdot)$ and then go to step 2.

α and β in equation (4.3) are two constants for determining the relative importance of the similarity measure and the contextual information. If the contextual information should be treated more important than the similarity measure, then β is set to a value greater than α and vice versa.

5. Experiments

Two arrays of data A and B, with size 150 each, are generated for testing the RFI algorithm. The mean and standard deviation of A are 198.8 and 20.9 respectively while the mean and standard deviation of B are 1016.1 and 36.0 respectively. A and B are concatenated together to model the gray level value distribution of two distinct regions in an image. Sixteen noise points with values 500 and 600 are added to A and B evenly. As shown in figure 1, the range 1 to 150 in x-axis represents the array of A while the rest represents B.

The RFI algorithm is employed to cluster the data into two sets. Initially, two clusters P and Q with center values 215.1 and 969.8 are set by calculating the mean of the noise corrupted data sets A and B. We assume that a data item can affect another data item only if they are neighbors to each other. In other words, W_{ij} is set to 1 if the two data items under consideration are neighbors. Otherwise, it is set to 0. The compatibility matrices C_1 and C_2 are set to $\begin{bmatrix} 0.3 & 0.0 \\ -0.7 & 0.3 \end{bmatrix}$ and $\begin{bmatrix} 0.3 & -0.7 \\ 0.0 & 0.3 \end{bmatrix}$ for data items in A and B respectively. Matrix indices i and j in $C_g(i,j)$ where g=1 or 2 correspond to the clusters P and Q respectively.

Figure 2a and 2b show the resulting membership function values of the data items to the final cluster centers 213.4 and 991.5 using RFI with α=0.9 and β=0.1 Figure 2c and 2d show the corresponding values to the final cluster centers 213.7 and 985.0 with α=0.3 and β=0.7. From these results, we observe that the attenuation is reduced when contextual information is taking into account during the clustering process.

6. Conclusion

In this research, the traditional fuzzy ISODATA (FI) algorithm is integrated with the probabilistic relaxation labeling (PRL) algorithm to form a new clustering algorithm called relaxed fuzzy ISODATA (RFI). During the clustering process, the data are grouped into some clusters based on both the distance metrics and the contextual information available. Further investigations are carried out in studying the convergence properties of the RFI algorithm. Also, a curve detection system will be developed as a test bed for this algorithm.

References

[1] Dave, R N, Use of the Adaptive Fuzzy Clustering Algorithm to Detect Lines in Digital Images, *SPIE Vol 1192 Intelligent Robots and Computer Vision VIII: Algorithms and Techniques* 600-611 (1989).

[2] Dave, R N, Generalized Fuzzy c-Shells Clustering and Detection of Circular and Elliptical Boundaries, *Pattern Recognition* Vol 25 No. 7 713-721 (1992).

[3] Krishnapuram, R, Hichem Frigui and Olfa Nasraoui, Fuzzy and Possibilistic Shell Clustering Algorithms and Their Application to Boundary Detection and Surface Approximation - Part I and II, *IEEE Transactions on Fuzzy Systems*, Vol 3 No. 1 Feb 29-60 (1995).

[4] Rosenfeld, A., Robert A. Hummel and Steven W. Zucker, Scene Labeling by Relaxation Labeling, *IEEE Transactions on Systems, Man and Cybernetics*, Vol. SMC-6 No. 6 420-433 (1976).

[5] Peleg, S., A New Probabilistic Relaxation Scheme, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. PAMI-2 No 4 362-369 (1980).

[6] Faugeras, O and M Berthod, Scene Labeling: An Optimization Approach, *Pattern Recognition* Vol 12 339-347 (1980).

[7] Haralick, R M, An Interpretation for Probabilistic Relaxation, *Computer Vision, Graphics and Image Processing* 22, 388-395 (1983).

[8] Kittler, J and J Foglein, On Compatibility and Support Functions in Probabilistic Relaxation, *Computer Vision, Graphics and Image Processing* 34, 257-267 (1986).

[9] Kittler, J and J Illingworth, Relaxation Labeling Algorithms - A Review, *Image and Vision Computing*, Vol 3 No 4 206-216 (1985).

[10] Zucker, S W, Robert A Hummel and Azriel Rosenfeld, An Application of Relaxation Labeling to Line and Curve Enhancement, *IEEE Transactions on Computers*, Vol 26 No. 4 334-403 (1977).

[11] Hancock, Edwin and Josef Kittler, Edge-Labeling Using Dictionary - Based Relaxation, *IEEE Transactions on Pattern Analysis and Machine Intelligence* Vol 12 No 2, 165-181 (1990).

[12] Christmas, William J, Josef Kittler and Maria Petrou, Structural Matching in Computer Vision Using Probabilistic Relaxation, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol 17, No. 8, 749-764 (1995).

[13] Kandel, Abraham, *Fuzzy Techniques in Pattern Recognition*, John Wiley & Sons (1982).

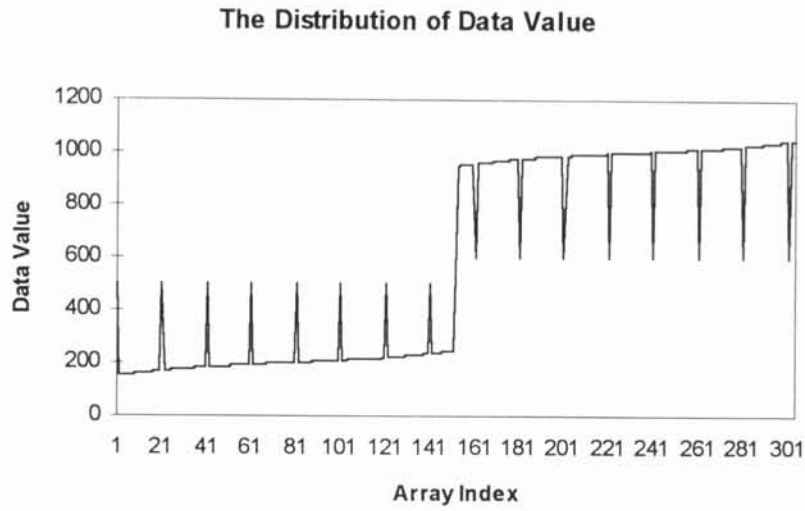

Figure 1. This figure shows the distribution of the testing data.

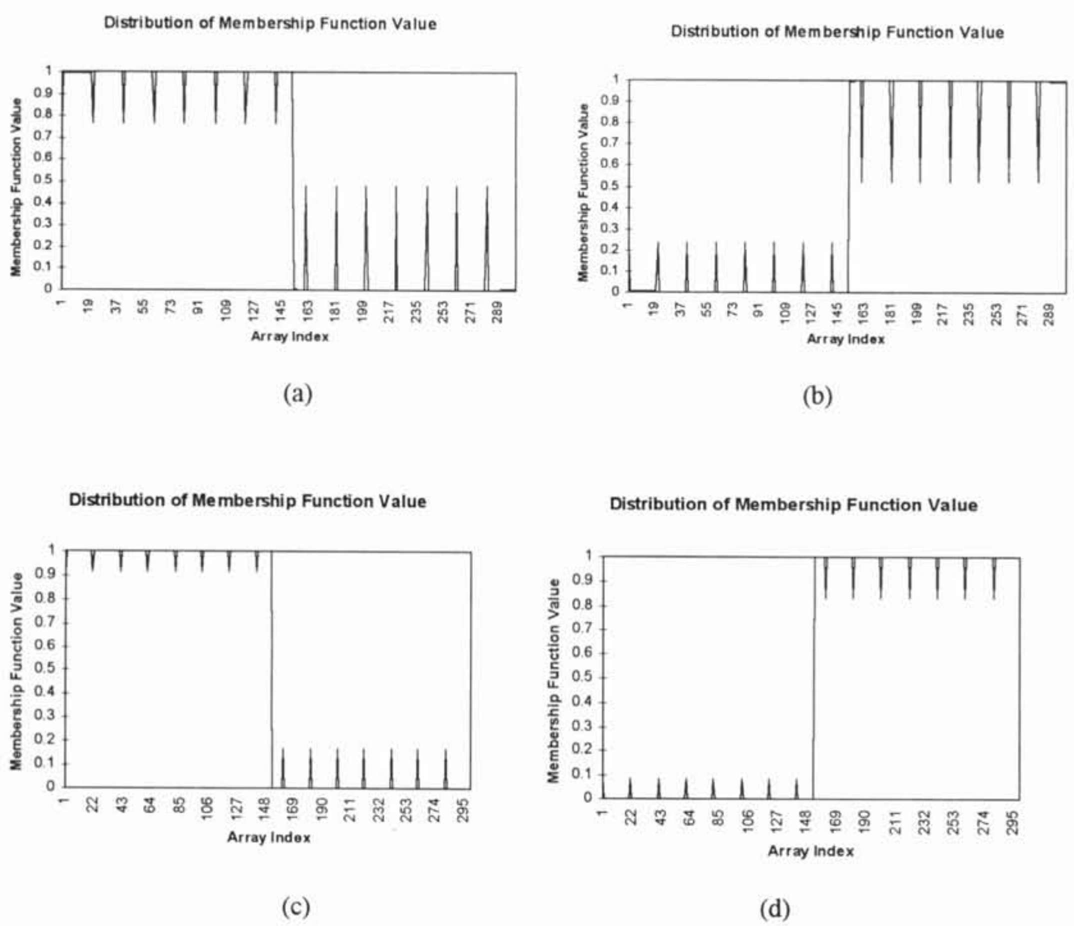

Figure 2. This figure shows the resulting membership function values using RFI with different α and β values. (a),(b) the membership function values of the data to the cluster centers 213.4 and 991.5 respectively with α=0.9 and β=0.1 (c),(d) the membership function values of the data to the cluster centers 213.7 and 985.0 respectively with α=0.3 and β=0.7.