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Planar Sharp Enhancement by General Geometric Heat Flow

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Abstract

In this paper, we want to find a corner enhancement curve deformation approach. We consider *General Geometric Heat Flow*(GGHF) as a general deformation approach, and the evolution of curvature in the approach is derived. The criteria of curvature scale space are proposed and subjected to the GGHF. The conditions under which the corner can be enhanced are also studied. From all these constraints, a new deformation scheme which can enhance strong corner and suppress noise and small structure while satisfies the scale space criteria is presented.

1 Introduction

Planar curve analysis is very important for object recognition and in the analysis, features extraction is a fundamental task. Among many features, such as arc, segment, corner singles out as the most important one. Usually, kernels should be used to extract such features, thus how to select appropriate scales for the kernels becomes a difficult but important problem. Inspired by the works of Witkin[18] and Koenderink[6] introducing scale space image processing, multiscale curve analysis(also known as curve-evolution) became very attractive for solving the key point[9][10][11][15][16][17]. A multiscale planar curve representation is very useful since it gives us a robust and analytical description of the information of the curve at different scales.

There are two widely used approaches to generate *Curve Scale Space*, namely *Gaussian Smoothing*[11] and *Curvature Deformation*[9][10]. Both of the two methods smooth curves so to suppress noise and weak features. Thus corners are also smoothed as the scale increases. In fact, as it was pointed out, the curve evolved according to curvature deformation would convergence to a "circle point"[3][4]. This is obviously a serious drawback when it comes to practical recognition since the salient features are destroyed in the course of evolution.

The problem we want to address here is to find other approach to construct the curve scale space, in which the strong corners(local curvature extrema with large curvature magnitudes)will be enhanced(the curvatures become bigger and bigger). However, any multiscale representation must satisfy some constraints or criteria so to guarantee that it is meaningful to the our purpose[19][14][2]. For example, when we performing corner enhancement, new corners should not be generated as we enhance the existed ones. This constraint should be satisfied by the scale space to be constructed in this paper. We generalize the criteria proposed for image scale space[19][14][2] to curve scale space and derive our approach under the condition that the criteria are satisfied.

2 Curvature evolution in GGHF

Consider a curve :

$$C_0(p) : S^1 \rightarrow R^2 \tag{1}$$

where S^1 donates the unit circle, and

$$C_0(p) = \{x_0(p), y_0(p)\}, \quad x_0(\cdot), y_0(\cdot) : S^1 \rightarrow R^1 \tag{2}$$

Usually, the curve is parametrized by a arbitrary parameter p while there exists a unique *Euclidean arc-length* parameter $s \in [0, L]$, $L=1$ is the length of the curve. $v = ((x_0)_p^2 + (y_0)_p^2)^{1/2}$ is the *Euclidean metric*, a characteristic function of the parameter p.

The curve evolves in time, where "time" represents scale. Let $C(p,t)$ donates the family of the deformed curves. In general, the curve deforms according to the General Geometric Heat Flow(GGHF):

$$\begin{aligned} \frac{\partial C}{\partial t} &= \beta\vec{N} + \alpha\vec{T} \\ C(p,0) &= C_0(p); \end{aligned} \tag{3}$$

We indicate with $\vec{N}$ and $\vec{T}$ the inward normal and the tangent unit vector of the curve. α, β are arbitrary functions. C_0 is the original curve. $C(p,t)$ is called the *Curve Scale Space*.

The *Curvature Deformation* is defined as:

$$\frac{\partial C}{\partial t} = k\vec{N} \tag{4}$$

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There are following formulas about the evolution of curvature k of the curve and its any order derivative in GGHF (See Appendix for proof):

$$\frac{\partial k}{\partial t} = \beta k^2 + \beta_{ss} + \alpha k_s \quad (5)$$

$$\frac{\partial}{\partial t}\frac{\partial}{\partial s} = (\beta k - \alpha_s)\frac{\partial}{\partial s} + \frac{\partial}{\partial s}\frac{\partial}{\partial t}$$

where we indicates with subscript the derivatives and the derivative with respect to t is obtained while keep p fixed.

3 Causality criteria in curvature scale space

What we are concerned with mostly for the curve is its shape, thus to construct a sample and efficient scale space, we should find a parameter to represent the shape at each point on the curve. Following[5], curvature singles out as a natural choice. So we transfer from the *Curve Scale Space* $C(p,t) : S^1 \rightarrow R^2$ to the *Curvature Scale Space* $k(p,t) : S^1 \rightarrow R^1$.

No spurious details should be generated passing from finer to coarser scale; In planar curve evolution, curvature zero-crossings(which separate the curve into convex and concave parts) and extrema(which represent the locations of the corners) are two most important kinds of features for characterizing the shape of the curve and are considered as "details" in curvature scale space. So new zero-crossing or extreme should not be generated as the scale increases. It was proved[10][16] that if there is:

$$\frac{k_{pp}}{k_t} > 0, \quad \textit{at the place where } k = k_p = 0 \quad (6)$$

no new zero-crossing(ZC) of the curvature will be generated; Similarly if there is:

$$\frac{k_{ppp}}{(k_p)_t} > 0, \quad \textit{at the place where } k_p = k_{pp} = 0 \quad (7)$$

no new extreme of the curvature will be generated. It is easy to prove that the two inequalities above are satisfied if and only if the following inequalities are satisfied respectively:

$$\frac{k_{ss}}{k_t} > 0, \quad \textit{at the place where } k = k_s = 0 \quad (8)$$

$$\frac{k_{sss}}{(k_s)_t} > 0, \quad \textit{at the place where } k_s = k_{ss} = 0$$

Now it is easy to prove following theorems:

Theorem 1 In the curvature scale space $k(s,t)$, if there is:

$$\frac{k_{ss}}{\beta_{ss}} > 0, \quad \textit{at the place where } k = k_s = 0 \quad (9)$$

then the evolution will not generate new curvature ZC.

Theorem 2 In the curvature scale space $k(s,t)$, if there is

$$\frac{k_{sss}}{\beta_s k^2 + \beta_{sss}} > 0, \quad \textit{at the place where } k_s = k_{ss} = 0 \quad (10)$$

the evolution will not generate new curvature extreme.

Since we do not know *a priori* where the point that $k = k_s = 0$ or $k_s = k_{ss} = 0$ will be, we ask the constraints to be satisfied at every point on the curve[6][10]. As we can see, α does not influence the *Causality* criteria, we can set $\alpha = 0$ or whatever appropriate for our special purpose.

If we select β to be a function of the curvature:

$$\beta = \beta(k) \quad (11)$$

From (9) and (10), it is easy to prove that no new ZC nor extreme will be generated if and only if :

$$\beta'(k) > 0 \quad (12)$$

where $\beta'(k)$ is the derivative of β with respect to k.

4 Corner enhancement

Usually corner is defined as the local extreme of curvature, i.e., $k_s = 0$, however for "real" or "meaningful" corners[14], there should be

$$kk_{ss} < 0 \quad (13)$$

which means only two special kinds of extrema are considered: positive maximum and negative minimum. Thus as the scale increases, if the corner is enhanced, there should be:

$$kk_t > 0 \quad (14)$$

Due to noise, we do not want to enhance all the extrema, only the extrema with large curvature magnitude should be enhanced, so there should be:

$$kk_t > 0, \quad |k| > TH \quad (15)$$

TH is a presetting threshold. The extrema with their curvature magnitudes smaller than TH should be smoothed so to suppress noise. Since any planar closed curve(convex[3] or un-convex[4]) evolved according to (4) will convergence to a "circular point", we ask that weak extrema should deform according to curvature deformation. While the corners are enhanced, the curvature of the points around the corner may also be enhanced. To guarantee that the corners are more and more distinguished from other structures as the evolution proceeds, the difference of the curvatures between the extrema and their neighborhood should increase which means the increase of the magnitude of k_{ss}. We have following criteria for corner enhancement:

1. At the extreme with $|k| \geq TH$, there should be $k_t k > 0$ and $k_{sst} k_{ss} > 0$.
2. At the extreme with $|k| < TH$, the curve should deform according to (4).

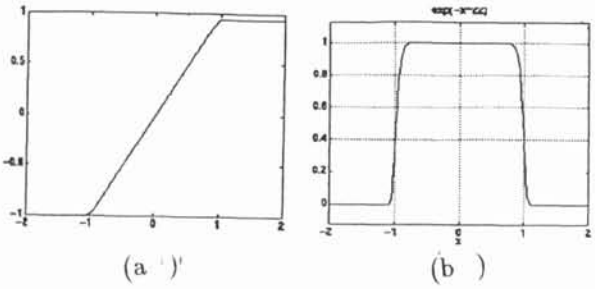

Figure 1: Deformation function $\beta(k)$ and its derivative when $\lambda = 20$ (a)$f(x)$, (b)$f'(x)$

5 Corner enhancement by GGHF

Now let's define a function $f(x)$:

1.
$$f(-\infty) = -f(\infty) < 0 \tag{16}$$

2.
$$f'(x) = e^{-(\frac{|x|}{K})^\lambda}, \quad K > 0 \tag{17}$$

where λ is a very big positive integer, for example, $\lambda = 20$. We have

$$f(-\infty) = -f(\infty) = -\frac{1}{2}\int_{-\infty}^{\infty} e^{-(\frac{|x|}{K})^\lambda} dx \approx -K \tag{18}$$

See figure 1.

Now let $\beta(k) = f(k)$, we have:

$$\beta(k) \approx \begin{cases} K, & k \geq K \\ k, & -K < k < K \\ -K, & k \leq -K \end{cases} \tag{19}$$

and

$$\beta'(k) \approx \begin{cases} 0, & |k| > K \\ 1, & |k| < K \end{cases} \tag{20}$$

and :

$$\beta'' \approx 0, \quad |k| > K \tag{21}$$

Other higher order derivatives of β with respect to k are also equal to zero when $|k| > K$. Since there is $\beta' > 0$, the curvature scale space criteria are satisfied. In the course of such evolution, no new corner will be generated, so we can concentrate our analysis on the existed corners. From (5) we have at the curvature extreme with $|k| > K$:

$$\begin{aligned} \frac{\partial k}{\partial t} &= \beta k^2 + \beta'' k_s^2 + \beta' k_{ss} \\ &\approx \beta k^2 = K|k|k \end{aligned} \tag{22}$$

And more:

$$\begin{aligned} \frac{\partial k_{ss}}{\partial t} &= \beta k k_{ss} + \frac{\partial}{\partial s}\frac{\partial}{\partial t} k_s \\ &= \beta k k_{ss} + (3\beta k k_s + \beta' k_s^2 + \beta_{sss})_s \\ &\approx 4\beta k k_{ss} \\ &= 4K|k|k_{ss} \end{aligned} \tag{23}$$

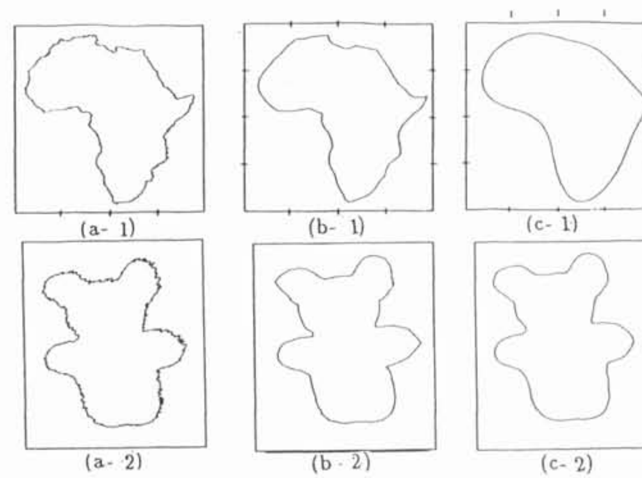

Figure 2: Corner enhancement. (a-1)Original curve 1, (b-1)Enhanced curve, (c-1)Curvature deformation. (a-2)Original curve 2, (b-2)Enhanced curve, (c-2)Curvature deformation.

The first enhancement condition are satisfied. The extreme with $|k| < K$ are deformed according to (4). Thus the constraints for both scale space and enhancement are satisfied.

Here we study some evolution when λ is applied to different values:

1. $\lambda = 1$(other smaller λ), then $f(x)$ is the ramp function. Here the threshold TH is no longer K, but the corners with their curvature magnitude $k > 3K$ are still enhanced, and other weaker corners have a similar behavior as curvature deformation.

2. $\lambda = \infty$, we have a optimal evolution for edge enhancement in which the approximate equations in this section become accurate equations. However this is a limited case which may violate the scale space criteria.

6 Experimental results

See Figure 2, the results from curvature deformation and our approach are compared. As we can, our approach enhance stronger corners and suppress noise.

7 Conclusion

We have combined Gaussian smoothing and curvature deformation into a general uniform geometric heat flow, from which, special evolution approaches for different purposes can be derived. To derive a corner enhancement deformation approach, we address two key points: first, Does it satisfy the scale space criteria? second: Does it enhances the strong corners? We present a new evolution approach which satisfies the two points and so can be used as pre-step for corner detection.

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Appendix

In this section we derive the formulas (5). Using the Frenet equations[2], we have:

$$\begin{aligned} \frac{\partial v^2}{\partial t} &= 2 < \frac{\partial C}{\partial p}, \frac{\partial}{\partial t}\frac{\partial C}{\partial p} > \\ &= 2 < v\vec{T}, \beta_p \vec{N} - \beta v k \vec{T} + \alpha_p \vec{T} + \alpha v k \vec{N} > \\ &= 2v(\alpha_p - \beta v k) \end{aligned} \tag{24}$$

So we have:

$$\frac{\partial v}{\partial t} = \alpha_p - \beta v k \tag{25}$$

and:

$$\begin{aligned} \frac{\partial}{\partial t}\frac{\partial}{\partial s} &= \frac{\partial}{\partial t}\left(\frac{1}{v}\frac{\partial}{\partial p}\right) \\ &= (\beta k - \alpha_s)\frac{\partial}{\partial s} + \frac{\partial}{\partial t}\frac{\partial}{\partial s} \end{aligned} \tag{26}$$

And:

$$\begin{aligned} \frac{\partial \vec{T}}{\partial t} &= \frac{\partial}{\partial t}\frac{\partial C}{\partial s} \\ &= (\beta k - \alpha_s)\vec{T} + \beta_s \vec{N} - \beta k \vec{T} + \alpha_s \vec{T} + \alpha k \vec{N} \\ &= (\beta_s + \alpha k)\vec{N} \end{aligned} \tag{27}$$

Let θ be the angle between the tangent vector and the x axis, then

$$\vec{N} = (-sin\theta, cos\theta) \tag{28}$$

$$\vec{T} = (cos\theta, sin\theta) \tag{29}$$

So:

$$\frac{\partial \vec{T}}{\partial t} = \left(-sin\theta\frac{\partial \theta}{\partial t}, cos\theta\frac{\partial \theta}{\partial t}\right) \tag{30}$$

So:

$$\frac{\partial \theta}{\partial t} = \beta_s + \alpha k \tag{31}$$

And we have:

$$\frac{\partial \theta}{\partial s} = k \tag{32}$$

We have:

$$\begin{aligned} \frac{\partial k}{\partial t} = \frac{\partial}{\partial t}\frac{\partial \theta}{\partial s} &= (\beta k - \alpha_s)k + \beta_{ss} + \alpha_s k + \alpha k_s \\ &= \beta k^2 + \beta_{ss} + \alpha k_s \end{aligned} \tag{33}$$