# Image Complexity Analysis for Self-Tuning Pattern Regeneration in Open Environment Knowledge Projection

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#### ABSTRACT

A non-deterministic image feature detection scheme is presented for interactive scene analysis. The image feature is identified with an attractor generated by a class of self-similar mappings. The mapping parameter is estimated through complexity analysis of non-linear diffusion field excited by observed imagery.

#### INTRODUCTORY REMARKS

Various decision support systems cooperatively generate environment description as the basis of schematic instruction [6], [7], [8]. Following computation model of cognition process [10], [12], the instruction schematics can be represented by a system of propositions defined on symbols deeply rooted in encountered environment. As the referents of propositions, the objects should be coded in terms of generic features. In articulating notyet-identified scene, on the other hand, the object should be coded in terms of observables. For denotatively preassigned objects, geometric models are available as feature representations: 3D contours as location invariants [4] and 2D grammar as phrase-structure invatiants [5]. However, morphological variations of object result in the Godel's trap [7]: The geometric model must be a priori adjusted to not-yet-encountered objects by an all-seeing-designer (Fig.1). In this paper, a non-deterministic scheme is introduced for object de-This scheme successively regenerates obscription. served pattern via the coordination of image complexity.



Fig.1. Gödel's Trap

#### NON-DETERMINISTIC OBJECT MODEL

Mathematically, this Gödel's trap is a paraphrase of the undecidability theorem [3]: For an arbitrary fixed algorithm  $\pi$ , there exists an observable and indicatable pattern  $\Lambda$  that is undecidable by  $\pi$ . Despite the intrinsic non-determinism, the detection scheme should be programmable within the framework of the formally closed systems: For a fixed set of observable-indicatable patterns ( $\Lambda_i$ , i=1,2,3...N), there exists an algorithm  $\pi$  for which arbitrary At. 1,2,3,...N, are decidable. To overcome this undecidability-programmability contradiction, the detection scheme invokes a non-deterministic description as an *a priori* object model. The basic idea of non-deterministic modeling is to describe the objects in terms of the invariant sets in joint iconic-symbolic feature space (Fig.2). In this description, the image feature is represented as a fractal attractor non-deterministically generated by a class of self-similar mappings [2]. The introduction of implicit representation implies that the contour patterns of not-yet-identified objects are anticipatively visualized *prior* to the completion of object modeling. In understanding an unstructured environment, the attractor model is combined with the ownership description [11] and the attractor of the motion [1] to generate an integrated a posteriori object description.



Fig.2. Non-Deterministic Object Model

## PATTERN REGENERATION PROCESS

Let  $\Lambda$  be an observation of an object contour via a dynamic version of the zero-cross scheme formulated by the following

$$\Lambda = \{ \Lambda \in \Sigma \mid |\Delta u| = 0 \text{ and } |\nabla u| > 0 \}, \qquad (1a)$$

$$\frac{\partial u}{\partial t} = \Delta u + \alpha [v - u], \ t \in T = [T_0, T_1],$$
(1b)

where  $\Sigma$  and v denote the image field and the gray level distribution in  $\Sigma$ , respectively. The response and resolution of observation  $\Lambda$  to object image v are simultaneously controlled by the positive parameter  $\alpha$ . When observed contour  $\Lambda$  is smooth, the pattern location, designated by  $\theta$ , is computed by the following

#### **Tracking Scheme:**

$$\frac{\partial \phi}{\partial t} = \Delta \phi - \gamma \phi, \phi = 1 \text{ on } \Lambda,$$
 (2a)

$$\frac{d\theta}{dt} = \int_{\Sigma} W \delta[\theta] \nabla \phi ds, \qquad (2b)$$

where  $\delta[\theta]$  and W denote Dirac's delta distribution and a properly chosen gain matrix [1]. The initial value of the location estimate  $\theta_0=\theta(T_0)$  is chosen as the minimal point of the diffusion field  $\varphi$ . For arbitrary  $\gamma$ -0, the detection scheme (2) subjected to smooth and convex pattern  $\Lambda$  yields unique minimal point  $\theta_0$ .

Consider a dissipative structure  $\sigma$  generated on irreversible thermodynamic system (2a) under the excitation of complicated pattern A. In this system, the energy flow  $q_{\sigma}$  is evoked between the excitation  $\chi[A]$  and the

heat sink (Fig.3). The control parameter  $\gamma$  in detection scheme (2) is adjusted so as to coordinate the complexity associated with fractal attractor  $\sigma$  and observable  $\Lambda$ .



Fig.3. Irreversible Thermodynamic System

#### Let $\Theta$ be the 2D distribution of the following

#### Null Entropy Generation Points:

$$\Theta = \{ \theta \in \Sigma \mid \nabla \ln \varphi \cdot q_{0} = 0 \}.$$
(3)

By definition, the distribution  $\Theta$  is a finitely extended version of the location  $\theta$  for generalized pattern  $\Lambda$ . The discrete distribution  $\Theta$  is specified in terms of the local minimum points without *a priori* information concerning not-yet-identified objects. Thus, we have the structural measure  $\Theta$  for *a posteriori* complexity evaluation of the observation  $\Lambda$ .

For regenerating the observation  $\Lambda$ , the discrete distribution  $\Theta$  is disintegrated via the following

#### **Field Interaction Scheme:**

$$\frac{\partial \phi}{\partial t} = \Delta \phi - (\frac{1}{\gamma})\phi, \quad \phi = 1 \text{ on } \{\Theta\}_{\tau} \cup \Xi_{\tau}[\Lambda], \quad (4)$$

for  $\tau \le t < \tau + 1$ . In Eq. (4b),  $\Xi_{\tau}[\Lambda]$  denotes the following

Equi-Field Set:

$$\Xi_{\tau}[\Lambda] = \{ \xi \in \Sigma \mid |\Delta \phi_{\tau}| > 0, |\Delta \phi_{\tau}| > 0, |\phi_{\tau} - \phi_{\tau}| = 0 \}.$$
 (5)

Noting that  $\Xi_{\tau}[\Lambda]$  converges to a self-similar approximation of  $\Lambda$ , define

$$\sigma = \lim_{\tau \to 0} \Xi_{\tau}[\Lambda]. \tag{6}$$

As a dissipative structure in non-linear diffusion system (4), the invariant pattern  $\sigma$  regenerates the observation  $\Lambda$ . Define

$$P(\sigma|\Lambda) = \frac{\int_{\Omega} \phi(s) ds}{\int_{\Sigma} \chi[\Lambda] ds},$$
(7)

for arbitrary regeneration  $\sigma$  and observation  $\Lambda.$  This  $P(\sigma|\Lambda)$  satisfies the following

## Properties of Conditional Probabilities:

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$$0 \le P(\sigma|\Lambda) \le P(\Lambda|\Lambda) \le 1,$$
 (8a)

$$P(\sigma|\cup\Lambda_i) = \sum P(\sigma|\Lambda_i), \ \Lambda_i \cap \Lambda_i = \emptyset, P(\Lambda_i) = P(\Lambda_i). \tag{8c}$$

Then, we have the measure  $P(\sigma|A)$  for a posteriori evaluation of the complexity of pattern regeneration process (5).

### STRUCTURAL COMPLEXITY ANALYSIS

Despite the non-anticipation, the discrete feature  $\Theta$  yields a cue to consistency evaluation of mapping candi-

dates. Let a class of self similar mappings  $\Pi = \{\pi_{I}, i=1,2,...\}$  be selected as a priori information. Then, the a posteriori consistency of the mapping  $\pi \in \Pi$  with attractor  $\Lambda$  is evaluated through self correlation analysis for the range of the projection  $\pi[\mathcal{D}[\Theta]]$ , where  $\mathcal{D}[\Theta] = \{\theta \in \Theta \mid \pi[\theta] \in \Theta\}$  denotes the domain of the mapping  $\pi$ .

First, the consistency of the a priori class  $\Pi$  is analyzed through the detection of the following

## Invariant Sub-class:

$$\Pi^{0} = \{ \pi^{0} \in \Pi \mid \exists_{\mathcal{D}^{0} \subset \mathcal{D}}[\Theta], \pi^{0}[\mathcal{D}^{0}] = \mathcal{D}^{0} \}.$$
(9)

Next, the collage theorem for the Iterated Function Systems [2] is invoked to estimate the correlation between the discrete patterns  $\Theta$  and  $\mathfrak{R}[\Theta] = \pi[\mathcal{D}[\Theta]] \cap \mathcal{D}[\Theta]$ , i.e., the restriction of the range of projection into itself. Then, the consistency of the mapping  $\pi$  is estimated based on the following

## Collage Error Evaluation:

$$h(\Theta, \mathcal{R}[\Theta]) \leq \frac{h(1 - C[\Theta])}{1 - L[\Theta]}, \quad (10a)$$

$$C[\Theta] = \frac{\|\mathcal{R}[\Theta]\|}{\|\mathcal{D}[\Theta]\|}, \quad (10b)$$

$$\mathcal{L}[\Theta] = \frac{\|\mathcal{R}[\Theta]\|}{\|\pi[\mathcal{D}[\Theta]]\|},$$
(10c)

where  $h(\bullet, \bullet)$  and  $||\bullet||$  denote the Hausdorff metric and size of the discrete pattern. In Eq. (10), C[ $\Theta$ ] and L[ $\Theta$ ] denote the coverage factor and the contractivity factor, respec-

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tively. Hence, a best fit mapping  $\pi^* \in \Pi^0 \subset \Pi$  is determined through a correlation computation on a finite pattern  $\Theta$ .

### PATTERN COMPLEXITY ANALYSIS

The adjustable parameter  $\gamma$  in pattern regeneration process (4) is controlled so as to coordinate the complexities associated with the approximation  $\sigma$  and the mapping  $\pi^*$ . This implies that, for adjusting the regeneration process (4), explicit specification of the mapping is not needed. The dissipative pattern  $\sigma$  well approximates the attractor  $\Lambda$  based only on the estimate of the "program length" for mapping description. The complexity associated with the non-deterministic regeneration process (4) is evaluated in terms of the probability of the dissipative pattern  $\sigma$  conditioned by the observation  $\Lambda$ . By

the estimation of the conditional probability  $P(\sigma|\Lambda)=$ 

 $\int \phi ds / \int \chi[\Lambda] ds$  and by applying Bayesian calculus to  $\sigma \sim \Sigma = \Sigma$  the following

#### the following

Fifty-Fifty Criterion:  

$$P(\sigma|\Lambda) = P(\neg\Lambda|\pi),$$
 (11)

we have a guideline for adjusting  $\boldsymbol{\gamma}$  in terms of the following

## Fixed Point Problem:

$$P(\sigma|\Lambda) = \frac{P(\pi)}{P(\Lambda)} [1 - P(\sigma|\Lambda)].$$
(12)

In Eq. (12) the ratio  $P(\pi)/P(\Lambda)$ , designated by relative complexity, indicates the description reduction of iconic pattern  $\Lambda$  by the constraint of mappings  $\pi$ . The relative complexity  $P(\pi)/P(\Lambda)$  is evaluated using the computational complexity  $\rho$  defined by the following

Complexity Coordination Rule:  

$$\rho = \min \left( \log_2 |\epsilon| - 1, \log_2 |\pi| \right), \quad (13)$$

where  $|\varepsilon|$  and  $|\pi|$  denote the length of error messages  $\varepsilon$ and mapping  $\pi$ . In Eq. (13), the error message is coded for specifying the  $\sigma$ - $\Lambda$  disparity independent of the location of  $\Theta$  [9]. Equation (13) implies that both overfitting mappings and too random deviations are rejected during regeneration. Thus, the computational complexity  $\rho$ provides the consistency evaluation of reasonable mappings  $\pi$  on the initial condition  $\sigma_0 = \Theta$ . Hence, the conditional probability P( $\sigma|\Lambda$ ) computed as the fixed point associated with the computational complexity  $\rho$ , yields the target for the diffused pattern  $\varphi$ . In other words, the control parameter  $\gamma$  is adjusted to reduce the error

$$\int_{\Delta} \frac{\sigma \sigma \Sigma}{\int_{\Sigma} \chi[\Lambda] ds}$$
(14b)

where  $\hat{P}(\sigma|\Lambda)$  denotes the solution to the fixed point problem (12) for a fixed relative complexity  $P(\pi)/P(\Lambda)$ . This implicit control process is formulated in terms of the following

#### Search Scheme:

 $\gamma \leftarrow \kappa^* \rho[1 - \lambda],$  (15a)

$$\frac{\gamma}{2} \leftarrow \kappa^*$$
. (15b)

Equation (14) successively updates the process parameter  $\gamma$  and associated conditional probability estimate  $\lambda \approx P(\sigma|\Lambda)$  simultaneously.

#### SIMULATION STUDIES

The pattern regeneration process is verified through a series of simulation studies. An example of simulation results is shown in Fig.4. In this simulation, a fractal pattern, "FERN", is generated by Monte Carlo simulation

and is regenerated through the proposed scheme. The observed fractal pattern A is well-approximated by the dissipative structure o generated by the non-linear diffusion system. In diffused pattern  $\phi$ , the distribution of null entropy generation points  $\Theta$  is detected. A set of mapping candidates was successively projected on this bottom up information  $\Theta$ -distribution. In the figure, the self similarity mapping  $\pi$  associated with the "FERN" pattern is successfully projected. Simultaneously, the disparity of the dissipative pattern  $\varepsilon$  and the length of the mapping  $\pi$  are evaluated to adjust the regeneration process to the target pattern  $\Lambda.$  As a result of this complexity coordination, the fit of the dissipative structure  $\sigma$ , the regenerated pattern, to the target attractor  $\Lambda$  is optimized so that the resulting conditional probability P(o[A) satisfies the fifty-fifty criterion.



(a) Encountered Pattern



(b) Mapping Selection Fig.4 Simulation Results

## CONCLUDING REMARKS

A non-deterministic detection scheme was presented for image features with self-similarity. In this scheme, the pattern to be detected is regenerated as the dissipative structure on non-linear diffusion field. The selfsimilar mapping is identified through the computational analysis of the null entropy generation points.

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