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USE OF THREE-DIMENSIONAL TEMPLATES FOR MULTIPLE SKELETONS

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ABSTRACT

We proposed a method for three-dimensional thinning using three-dimensional templates developed for the robust operation. Thinnings proposed so far extracted only one skeleton from the object. Our result gives multiple skeletons, each of which fulfills the three conditions for the skeleton. They are: preserving connectivity, a one-voxel thickness, and no change in the endpoints. We proposed some two hundred $3 \times 3 \times 3$ templates for the thinning with capability of preservation of connectivity and the endpoints. Seventy-two $2 \times 2 \times 2$ patterns to check the one-voxel thickness are also proposed. Examples of three-dimensional primitive of a cube and a space shuttle model are demonstrated. We compared these multiple skeletons from the standpoint of closeness to the medial axis and the location of gravity centers.

INTRODUCTION

Thinning is a sequential process which shrinks an object to a skeleton, fulfilling the generally adopted three conditions; preserving connectivity, extracting a one-voxel thick figure, and unchanging endpoints. For the thinning, most previous efforts were limited to two-dimensional algorithms. For the three-dimensional thinning some five studies have been reported. They are theoretical [1,2,3] or give no validated examples [4,5]. Also all of them are limited to extracting only one skeleton from the object. In our method, multiple skeletons are obtained from the same object using some twelve templates. An optimal skeleton is discussed based on the medial axis and the center of gravity (centroid).

TEMPLATES FOR THE THREE-DIMENSIONAL THINNING

Thinning is a process which obtains skeletons in order to remove voxels from the boundary of an object. In the process, the above three conditions should be fulfilled. A voxel whose deletion preserves connectivity and does not change endpoints is called a deletable point. Thinning is realized to sequentially remove these deletable points. Whether a voxel is deletable or not, that is, the connectivity is preserved or not, and the point is the endpoint or not, is determined by checking its 26-neighbors which are involved in a $3 \times 3 \times 3$ region. We developed adequate templates in this region to check the 26-neighbors. If a pattern around a scanning point in the object matches a template centered on the point, the point is said to be deletable. When black and white points in a pattern coincide with the respective black and white points in the template, the pattern is said to matche the template. Here, "black point" is the point which has a value of '1', and the "white point" is the point which value has a value of '0'.

Usually, an object is treated such that it consists of 26-connected black points and its complements consist of 6-connected white points. If this condition is not fulfilled, the object is divided into two or more parts. If a black point is deleted and the object still remains undivided, it is said that the connectivity is preserved. When connectivity is preserved, the resulting skeleton is not divided into two or more segments.

The endpoint is defined as the point which has only one point in its 26-neighbors. If it is an endpoint, there are two black points including itself in a $3 \times 3 \times 3$ region. Therefore, when a template has two black points, the endpoint may be deleted. To avoid this, a template has more than three black points.

We defined templates in order to fulfill these two conditions. They are shown in Fig. 1. For example, template T_1 has four 26-connected black points and seven 6-connected white points. Even if the center voxel of this template is changed to a white point, the connected black points are not divided into two or more parts. Since this template consists of four black points, endpoint deletion does not occur.

THREE DIMENSIONAL THINNING PROCESS

An example of the thinning process is shown in Fig. 2.

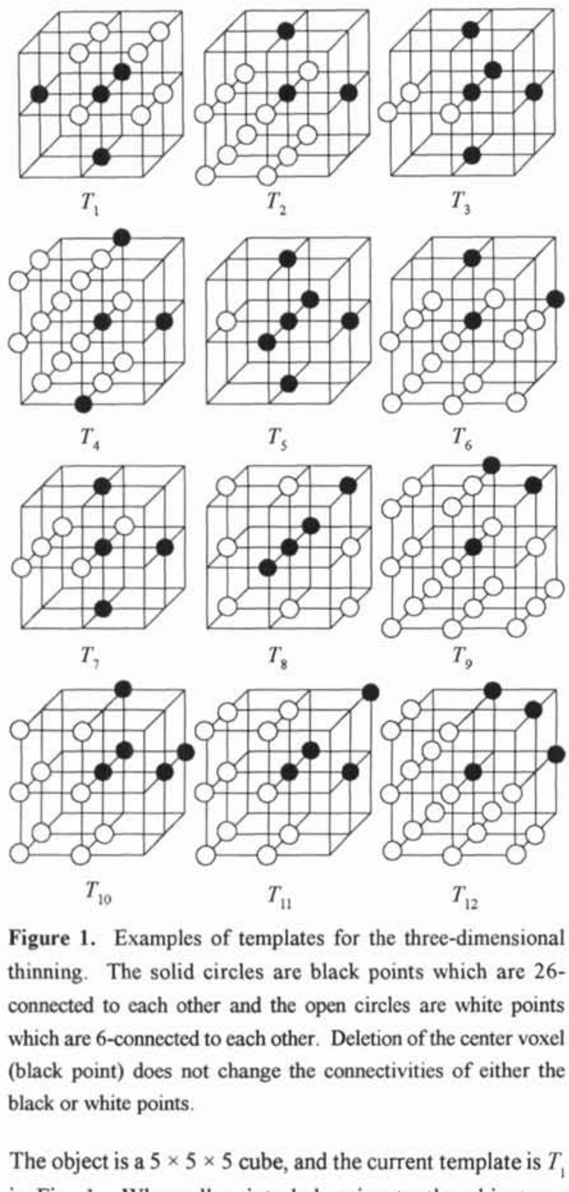


Figure 1. Examples of templates for the three-dimensional thinning. The solid circles are black points which are 26-connected to each other and the open circles are white points which are 6-connected to each other. Deletion of the center voxel (black point) does not change the connectivities of either the black or white points.

The object is a 5 × 5 × 5 cube, and the current template is T_1 in Fig. 1. When all points belonging to the object are checked using template T_1, only point a is said to be deletable.

After checking all points using one template, deletable points are removed. The next checking process then operates using another template. If the same template is consecutively used for the next process, the object shrinks in one direction and a bias skeleton results. To avoid a bias skeleton, the next template should be a symmetrically rotated figure of the first template.

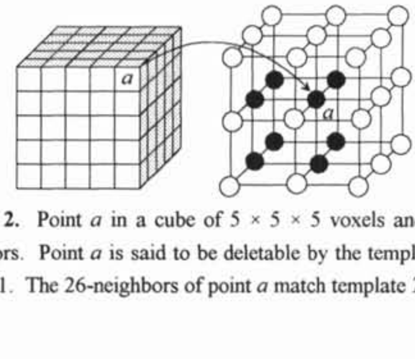
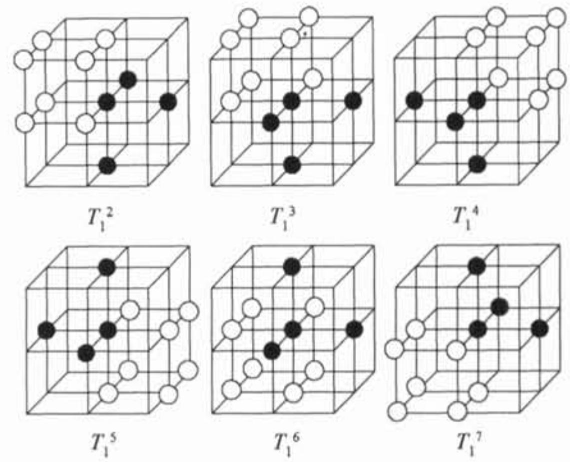


Figure 2. Point a in a cube of 5 × 5 × 5 voxels and its 26-neighbors. Point a is said to be deletable by the template T_1 in Figure 1. The 26-neighbors of point a match template T_1.



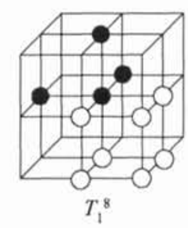


Figure 3. All possible symmetrically rotated templates T_1^2, ..., T_1^8 of the template T_1.

A three-dimensional template has 6, 8, 12 or 24 symmetrically rotated templates. When a template fulfills the above two conditions, each of the rotated templates fulfills the same conditions. In Fig. 3, template T_1 in Fig. 1 and all its possible symmetrically rotated templates, T_1^2, ..., T_1^8, are shown. Following the use of T_1, all symmetrically rotated templates T_1^2, ..., T_1^8 should be sequentially used. The results of the removing process using these eight templates are shown in Fig. 4(a). The set of templates $\{T_1\}=\{T_1, T_1^2, ..., T_1^8\}$ is used for one sequential operation. The second

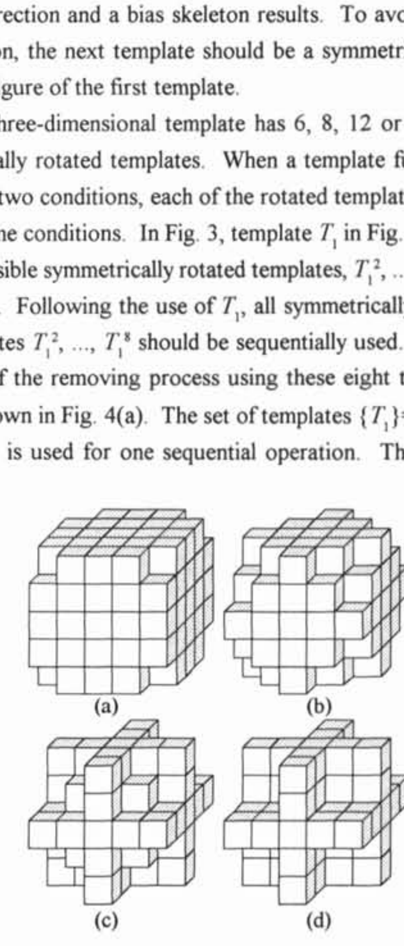


Figure 4. Intermediate figures (a), (b) and (c) in the thinning process using the set of templates $\{T_1\}$. No further changes occur in the following process. This is the converged result (d).

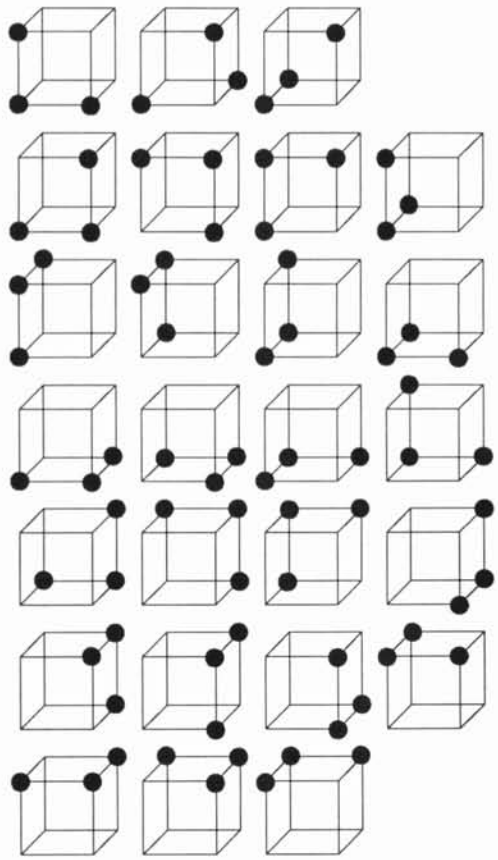


Figure 5. Three fundamental patterns (top three) to check of one-voxel-thickness. They are independent of each other. Each of them has twenty-three symmetrically rotated patterns. Symmetrically rotated patterns of the first fundamental pattern are shown from the second row. All seventy-two patterns are used for this purpose.

result of the second operation is shown in Fig. 4(b), the third is in Fig. 4(c), and the fourth is in Fig. 4(d). Further operations remove no more voxels, so the fourth result is said to have converged in this case.

The converged result using a set of templates is not always one voxel thickness as shown in Fig. 4(d). To obtain one voxel thickness figure, another set of templates is to be used for the next operation. In general, if a one voxel thickness figure is not obtained, further operations are required by changing the set of templates.

To check whether one voxel or not, the three 2 × 2 × 2 patterns shown in Fig. 5 and their symmetrically rotated patterns are used. When these patterns are included in an intermediate result, it is not a one voxel thickness.

Our thinning process is summarized in Fig. 6. Multiple skeletons are obtained using the sequences of sets of templates.

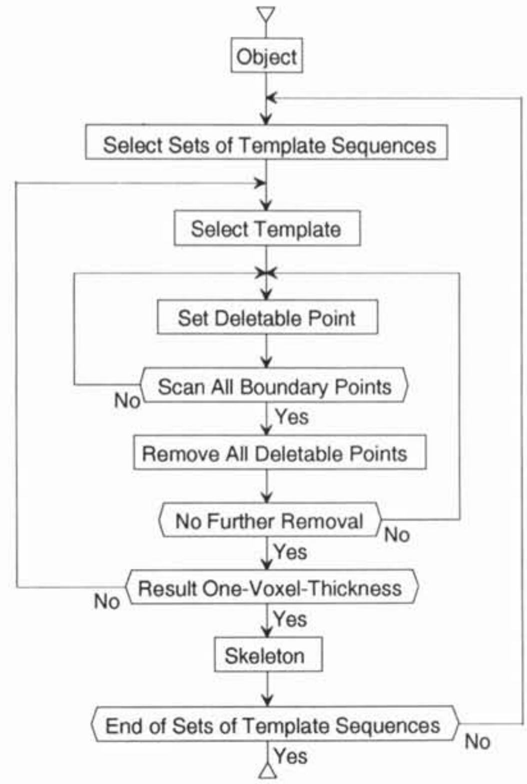


Figure 6. Flow chart of the thinning process for multiple skeletons. We prepared twelve sets of templates and seventy-two one voxel checking patterns.

EXAMPLES OF MULTIPLE SKELETONS

As an example notation, the process using four sets of templates $\{T_1\}$, $\{T_2\}$, $\{T_3\}$, $\{T_4\}$ is expressed as $\psi(T_1, T_2, T_3, T_4)$.

An object of a cube of 33 × 33 × 33 voxels and its three skeletons are shown in Fig. 7. The skeletons are obtained

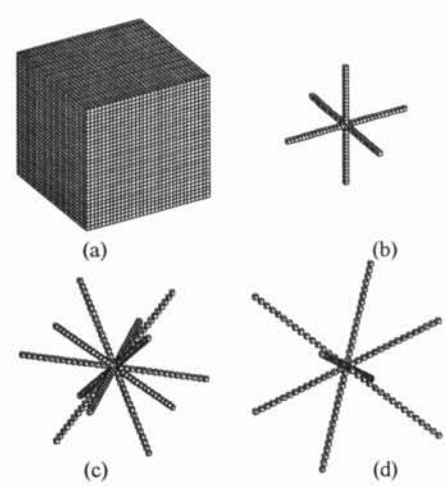


Figure 7. An object of a cube of 33 × 33 × 33 voxels (a), and three skeletons (b), (c), and (d).

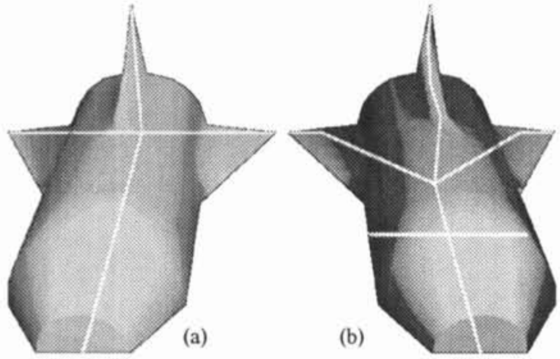


Figure 8. An object of a space shuttle model and two skeletons (a) and (b).

by the processes, $\psi(T_1, T_2)$, $\psi(T_1, T_5, T_7)$ and $\psi(T_3, T_4, T_9)$. The three skeletons (b), (c), and (d) consist of 6-, 18-, and 26-connected lines, respectively. The skeleton (b) is simple and same as the Cartesian coordinate. The skeleton (c) crosses the edge centers, whereas the skeleton (d) is diagonal.

A space shuttle model and its two skeletons are shown in Fig. 8. Skeletons are obtained by the process $\psi(T_1, T_2)$ and $\psi(T_1, T_{10}, T_2)$. The skeleton (a) is simple but has no information about the shape of thr wings, however, skeleton (b) does indicate this fact.

SKELETON FEATURES

Different skeletons are obtained by changing the order and the combination of the set of templates in the thinning process. We define two criteria to evaluate the multiple skeletons. They are the medial axis and the location of the gravity center. The skeleton, which is closest to the medial axis and has the closest location to the gravity center, is determined to be the best.

The medial axis is obtained using Medial Axis Transformation [6]. Let S_0 is the medial axis of three-dimensional object, and S_i is ith skeleton of multiple skeletons. Numbers of voxels in three-dimensional shape X is denotes as $N(X)$. We defined a closeness factor λ between the medial axis and a skeleton as follows:

$$\lambda_i = \frac{N(S_0 \cap S_i)}{N(S_0 \cup S_i)}$$

A skeleton which has more voxels in the medial axis has a larger value of λ. The value λ ranges from 0 to 1. Here, $\lambda = 1$ indicates that a skeleton S_i is identical to the medial axis S_0. Usually, a three-dimensional medial axis consists of planes or some divided figures. So the medial axis itself is not a skeleton. A skeleton is desired to be similar to the "ideal medial line." We compared skeletons with the medial axis instead of the "ideal medial line."

We proposed another criterion using the gravity center. If a skeleton is bias, the gravity center of it is different from that of an object. When the gravity center of an object is g_0, that of ith skeleton is g_i and the longest distance among the endpoints in the skeleton is l, we defined a factor δ as follows:

$$\delta_i = 1 - \frac{\overline{g_i g_0}}{l}$$

The value δ ranges from 0 to 1. Here, $\delta = 1$ indicates a skeleton and the object has the same gravity center.

We evaluated skeletons using a following criterion C.

$$C_i = \lambda_i + \delta_i$$

For a cube, previously reported skeletons are similar to that of Fig. 7(b), however, our best skeleton from the criterion is that of Fig. 7(d). The both figures have the same gravity center, however, our skeleton is identical to the medial axis, which is identical to the medial line in this case. For the space shuttle model, the skeleton of Fig. 8(c) is concluded better.

As described avobe, our thinning process obtains inherently multiple skeletons. This will be of use for more accurate shape recognition.

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