

# A TECHNIQUE FOR RECONSTRUCTING SHAPE OF SPECULAR SURFACES

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## ABSTRACT

Various techniques for measuring 3-D shape of an object which use optical information have been developed. But the object is assumed to have diffuse surfaces in most studies. In this paper, we describe a technique for reconstructing 3-D shape of objects which have specular surfaces based on geometric relations between positions of point light sources and directions of the light reflection from surfaces. In addition, we present a measure for evaluating the error of reconstruction. Also we propose a new range finding method which enables us to obtain shape of a specular surface directly. We have developed a 3-D measuring system, and demonstrate that the shape of relatively large-sized specular surfaces can be reconstructed with satisfactory accuracy.

## INTRODUCTION

Various noncontact range finding techniques have been developed. Multiple views, a slit ray projection or moire fringe analysis are well known, and some systems for measuring 3-D shape have been put into practical use. On condition that incident light is scattered by surfaces, range data can be obtained by triangulation techniques using a structured light and an image sensor. But for objects which have specular surfaces, glass or mirror finishing steel for example, such triangulation techniques can hardly be applied. The reason is that because a narrow beam of light projected to the surfaces is reflected such that the angle of reflection equals the angle of incidence, the reflected light does not always reach the sensor. If we intend to detect the reflected light at all times, we should adjust the position and orientation of the sensor, the light source or the surfaces, and then we are obliged to build a complicated measuring system.

Some studies of determining shapes of specular objects have been done [1]-[4]. However, since an object is assumed to be a body of revolution or a polyhedron, the shape is not so arbitrary in some cases. While developed techniques are useful to recognize shape of an object, the accuracy of reconstruction is not discussed in other cases. In this paper, we describe a developed 3-D measuring system, and demonstrate that the shape of 33" CRT panel is reconstructed with satisfactory accuracy using the developed technique. Also we

propose a new range finding method which enables us to obtain shape of a specular surface directly.

## SHAPE RECONSTRUCTION

Our reconstruction procedure is divided roughly into two processes. The one is optical examination which consists of two steps. The first step is to determine from what direction light reaches each pixel of an image which is obtained by a fixed TV camera. Light emitted from a point light source is reflected by a flat mirror opposite to the source, as shown in Fig. 1. Now we consider one of the pixels and move the source in parallel with the mirror so that the light reaches the pixel. Since the position of a virtual image of the source can be found easily and a lens center is given, we can determine the direction through the pixel and the lens center. We will call it the viewing direction from now. The second step is to determine at what position the source is located when the light reaches the pixel after it is reflected by a specular surface, as shown in Fig. 1.

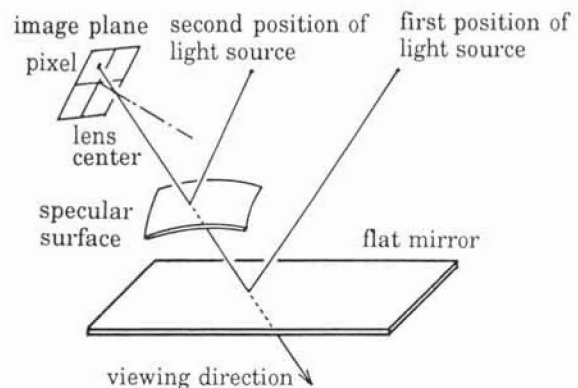


Fig. 1 Optical examination.

The other process is shape reconstruction. Fig. 2 shows geometric relation between the viewing direction of a certain pixel and its related position of the source. As seen in the figure, infinite number of surfaces can meet the condition of reflection. Now we define an orthogonal coordinate system  $o-uv$  in a plane which is defined by the lens center, the viewing direction and the position of the source.

Normal vector of the surface lies in the plane. Let  $v = au$  denote an equation of a line along the viewing direction, and  $S(u_s, 0)$  the position of the source. Then the tangent  $t$  of a section of the curved surface at a point  $P(u,v)$  where the light is reflected is expressed in an equation,

$$a(2u - u_s) t^2 + 2(u(1 - a^2) - u_s) t - a(2u - u_s) = 0$$

It shows that  $t$  is a function of  $u$ . If one point on a local tangential plane is given, the position and orientation of the plane is determined.

Fig. 3 shows the procedure of reconstruction. Initially the viewing direction  $V_1$ , the position of the source  $S_1$  and a constrained point  $Q_1$ , which is called the initial constrained point from now, are given. We will describe how to get  $Q_1$  later. At first the tangent of the local small plane  $\pi_1$  is calculated so as to meet the geometric relation of specular reflection. Next the position and the orientation of  $\pi_2$  is determined similarly. On this occasion,  $Q_2$  is defined by iterative calculation so that distance from  $Q_2$  to  $P_1$  equals that from  $Q_2$  to  $P_2$ . In such manner the point  $P_i$  where the light from  $S_i$  is reflected is determined one after another. As a result, we obtain the same number of points as that of pairs of the viewing direction and the related position of the source. The surface of the object is reconstructed with the set of these points.

Though the point has three dimensional coordinates, it can be considered in the two dimensional plane denoted  $o-uv$ . After the coordinates are defined locally, they are transformed to a world coordinates. Because the next local plane  $\pi_{i+1}$  is determined only by the pair of  $V_{i+1}$  and  $S_{i+1}$ , the result of calculation depends on a sequence in which the pair is selected, provided that the object is a flat surface or a sphere, the curvature of which is constant. But the maximum displacement of the reconstructed points due to the difference of sequence is not greater than  $5\mu\text{m}$  in regard to the experimental result detailed later. Judging from the required accuracy, the value is negligible. However, the way of calculation can be improved.

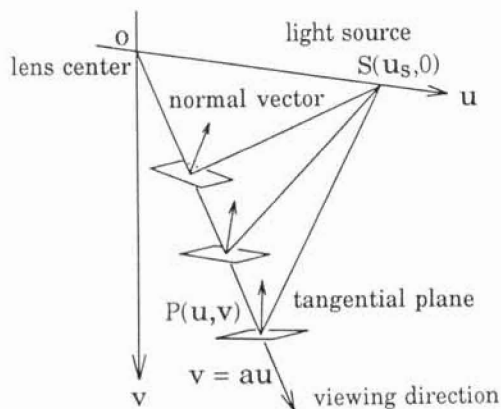


Fig. 2 Geometric relation between the viewing direction and the related position of the source.

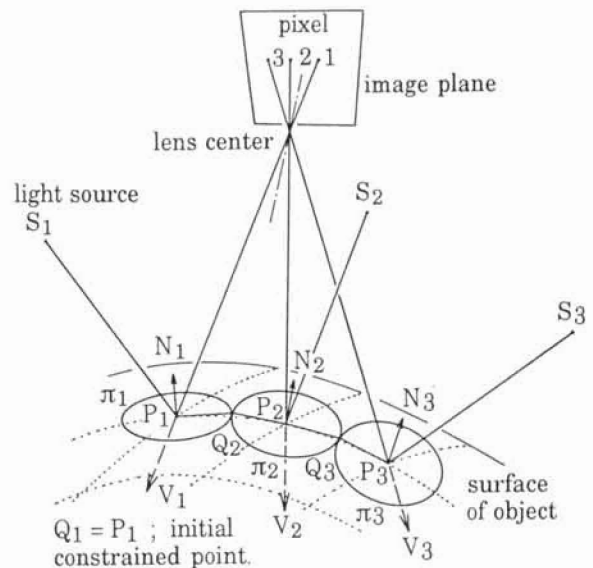


Fig. 3 Procedure of surface reconstruction.

### RANGE FINDING

As stated above, the initial constrained point is indispensable to reconstruct the shape. We can get 3-D coordinates of a point on the surface with a 3-D measuring machine. If the point is allowed to be marked, its 2-D position in the image is easily determined, and the calculation can be started. But it is not practical method.

Now we propose a new range finding method using two cameras. First of all the optical examination with each camera is done. Fig. 4 shows the

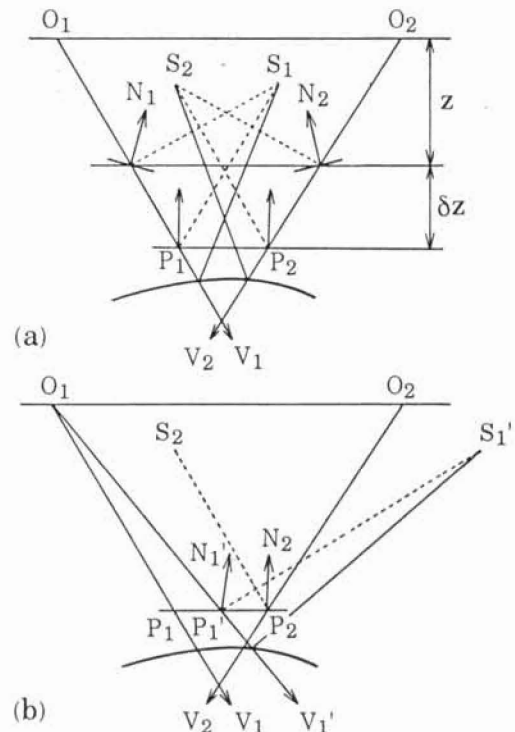


Fig. 4 Principle of the range finding method.

principle of the method. Initially the viewing directions  $V_1, V_2$ , the related positions of the light source  $S_1, S_2$  and depth  $z$  is given anyway. Then normal vectors  $N_1, N_2$  of tangential plane of a surface are determined on the geometric relation of reflection. Let  $P_1$  and  $P_2$  denote an original point of the vectors. If the true depth of  $z$  is given, the two conditions, namely  $N_1$  is almost parallel to  $N_2$  and the distance  $P_1$  to  $P_2$  is very short, should be satisfied. So at first the depth  $z$  is increased or decreased gradually according to change of the angle between  $N_1$  and  $N_2$ , until  $N_1$  is parallel to  $N_2$ . After that however,  $P_1$  does not coincide with  $P_2$ , because two lines along  $V_1$  and  $V_2$  do not intersect at the surface of the object. So one of the viewing direction is modified successively as shown in Fig. 4(b). New viewing direction  $V_1'$  is taken toward the middle point of a line segment  $P_1P_2$  denoted  $P_1'$  and the related position of the light source is shifted to  $S_1'$ . In this manner  $z$  and  $V_1$  is changed alternatively until the two conditions are satisfied simultaneously. Consequently the true depth is obtained.

### LIGHTING AND DETECTION

Fig. 5 shows configuration of the optical measuring system. A coordinate system denoted  $o-xyz$  is defined in the measuring space. Laser scanner mechanically draws a bright line on a screen parallel to the  $x$ -axis. The system has other scanner which draws a line parallel to the  $y$ -axis. We regard a intersection of two bright lines perpendicular to each other as a point light source expressed in the previous section.

As stated before, the foundation of our technique of reconstruction is to determine at what position the light source is located when light from the source reaches a certain pixel. So it is very important to detect its time with high accuracy. When a bright line parallel to the  $y$ -axis sweeps in the direction of the  $x$ -axis, the time can be

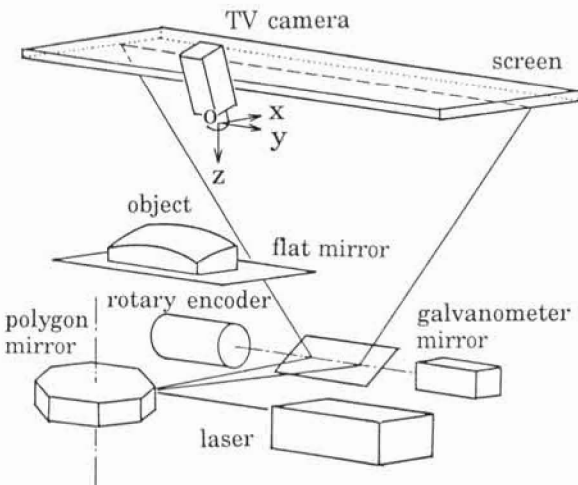


Fig. 5 System configuration.

determined from the brightness curve of the pixel, assuming that the brightness reaches its peak when an image of the line passes at the center of the pixel [5]. The  $x$  coordinate of the bright line can be calculated from an output value of a rotary encoder with which the axis of galvanometer mirror is equipped. By sweeping a line parallel to the  $x$ -axis, the  $y$  coordinate is obtained similarly.

### EVALUATING THE ACCURACY

In order to evaluate the accuracy of reconstruction, we get 3-D coordinate of points on the surface with a 3-D measuring machine, and generate spline curve surface. Regarding it as a master shape, we compare the shape obtained by our technique with it. Since the two shapes are defined with different coordinate system, we should unify the systems before comparison. If points  $P_i$  and its identical points  $Q_i$  ( $i = 1, n$ ) are given in these two coordinate systems respectively, transformation matrices can be found by using a method of least squares [6].

Because the information input for reconstructing shape are expressed in the coordinates of  $o-xyz$ , it ought to be considered that the coordinates of the reconstructed points represent the real position of the surface. But in practice the whole of the reconstructed points is rotated by a very little angle owing to the error of the measured data or device calibration. Therefore before two shapes are compared, the reconstructed one is rotated, assuming that the axis of rotation passes through the initial constrained point and is parallel to  $xy$  plane. Let  $\omega$  and  $(l, m, 0)$  denote the angle and the axis of rotation, and  $\delta z_i$  the distance from the  $i$ -th reconstructed point to the master surface as shown in Fig. 6. The angle  $\omega$  and the components of the vector  $(l, m, 0)$  are determined such that sum of squares of the distance is minimized after rotation. At first  $\delta z_i$  is evaluated. When the angle of rotation is sufficiently small, the distance  $\delta z_i'$  which is evaluated after rotation is expressed by,

$$\delta z_i' = m \omega x_i - l \omega y_i + \delta z_i$$

Let  $E = \sum \delta z_i'^2$ , and solving  $\partial E / \partial l = 0$  and  $\partial E / \partial \omega = 0$ ,

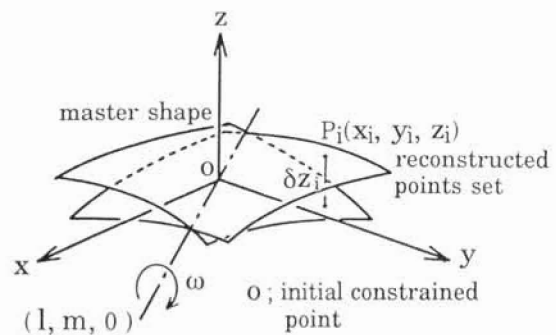


Fig. 6 Rotating the surface.

then we obtain,

$$\omega = \frac{l T_{yz} - m T_{xz}}{m^2 T_{xx} - 2lm T_{xy} + l^2 T_{yy}},$$

$$l = \frac{\alpha}{(\alpha^2 + \beta^2)^{1/2}}, \quad m = \frac{\beta}{(\alpha^2 + \beta^2)^{1/2}},$$

where,

$$T_{xx} = \sum_{i=1}^n x_i^2, \quad T_{yy} = \sum_{i=1}^n y_i^2, \quad T_{xy} = \sum_{i=1}^n x_i y_i,$$

$$T_{xz} = \sum_{i=1}^n x_i \delta z_i, \quad T_{yz} = \sum_{i=1}^n y_i \delta z_i,$$

$$\alpha = T_{xy} T_{xz} - T_{xx} T_{yz}, \quad \beta = T_{yy} T_{xz} - T_{xy} T_{yz}$$

As a result we can rotate the points and compare the two shapes properly.

### EXPERIMENTAL RESULTS

In this section we show experimental results. The object is a 33" CRT panel. The image is 256 x 256 pixels. A measured area for reconstruction is limited to middle part of the panel. The area is about 300 x 300 mm, and radius of curvature is about 2,000 mm and 1,000 mm in horizontal and vertical section respectively. The coordinates of the initial constrained point is measured with the 3-D measuring machine and is transformed to those of o-xyz. Fig. 7 shows a wire frame of the object. It is generated by connecting every 5 reconstructed point, the z-coordinate of which is magnified by 5 times.

Fig. 8 shows the distribution of deformation of the shape from the master one with gray scale. The difference between the maximum and minimum deformation is 178 μm, which is 798 μm evaluated before rotation. The angle of rotation is 1.6 x 10<sup>-3</sup> rad. The result proves that the shape of the object is reconstructed with high accuracy.

The result of the range finding is that the difference between the two measured values, obtained by a contact method and by our range finding method is 0.49 mm. Theoretically the whole shape of a specular surface is known by applying our method to the whole surface, if the accuracy is improved.

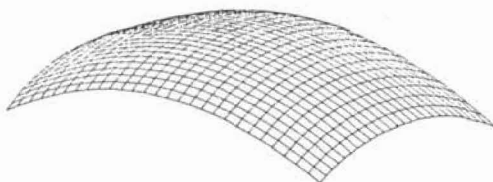


Fig. 7 A wire frame of the object.

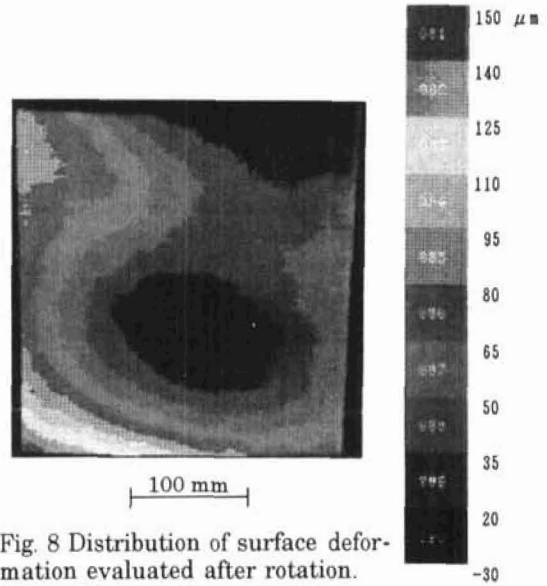


Fig. 8 Distribution of surface deformation evaluated after rotation.

### CONCLUSION

We developed a technique for reconstructing a shape of specular surfaces, and we demonstrated that the shape of 33" CRT panel is reconstructed with high accuracy. Moreover we proposed a new range finding method for getting the depth of specular surfaces using multiple cameras. Since almost all studies have been interested in diffuse surfaces, our technique is valuable.

We obtained good results for the present. However there are several subjects for future study. It is necessary

1. to investigate the cause for rotation of the whole reconstructed shape.
2. to improve the sequence of calculation for reconstruction.
3. to settle problems about reflection from the reverse side of glasses.
4. to apply the technique to a larger object.

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