REPRESENTATION AND RECONSTRUCTION OF THREE–DIMENSIONAL OBJECTS USING NONLINEAR DEFORMABLE SUPERQUADRIC MODELS*

Chao Zhang and Baozong Yuan
Northern Jiaotong University

Beijing, 100044, P.R.China

ABSTRACT

We propose a new kind of models, which are called Nonlinear Deformable Superquadric (NDS) models, for representing and reconstructing 3D objects. Instead of linear tapering deformation, a piecewise cubic Bezier spline is used to deform superquadrics. By changing the control points of the spline, the shape of the model is deformed as if it was made of a flexible material. In this manner, we can uniformly handle a large set of objects or parts with a small set of parameters. We developed an algorithm for NDS model reconstruction from single monocular image. We first capture the global shape features of the object by extracting the size of a superquadrics. And then an adaptive algorithm automatically fits a piecewise Bezier curve to the image contour. Using the spline to deform the superquadrics, a 3D object with irregular, local shape features is reconstructed. Experimental results using real images show that the reconstructed models are well conformed to the data and the reconstruction procedure is fast.

1 INTRODUCTION

The problem of representing and reconstructing three dimensional (3D) objects has received an enormous amount of attention in computer vision research for the past decade. However, it is still an open problem on how to select an appropriate model in order to meet the requirements of both object reconstruction and object recognition.

There is a serious conflict between the requirements of reconstruction and recognition. General–purpose reconstruction in low level visual processing requires models with broad geometric coverage. Reconstruction models must extract detail information from noisy sensor data while making the weakest possible assumptions about observed objects. By contrast, object recognition is a higher level process that necessitates drastic information reduction and shape abstraction in order to support efficient matching in object databases of manageable size.

Generalized spline models appear to be well suited to object reconstruction. They are free–form because their local shape control variables provide many local degrees of freedom[5]. Consequently, splines have the flexibility to represent diverse shapes, i.e., they have broad geometric coverage. But generalized spline models require too much data storage for object recognition. Volumetric primitives such as spheres, cylinders, and prisms seem appropriate for object recognition since they can decompose composite shapes into natural parts that are compactly expressible using a small set of parameters.

Recently, a new modeling method , which is called superquadrics, has been used to describe smooth objects in computer vision[7] and in computer graphics[1]. Superquadrics can model a large family of standard shapes, like cubes, cylinders, spheres, diamonds and pyramidal shapes as well as the round–edged shapes intermediate between. They can also be deformed to many irregular classes of objects.

In this paper, we propose a new model which is called "Nonlinear Deformable Superquadric (NDS) models". Instead of the linear tapering deformation we use a piecewise cubic Bezier spline for deformation. Given a set of contour points, the system can automatically generate a piecewise cubic Bezier curve. By changing the control points, the shape of the model is deformed as if it was made of a flexible material. In this manner, we can uniformly handle a large set of objects with a small set of parameters. NDS model considerably increases the geometric coverage of superquadrics. It simultaneously satisfies the requirements of reconstruction and recognition and promises a fluent transition between these two aspects of vision.

The configuration of this paper is to begin with a description of the NDS model. We then describe the model reconstruction process, which consists of a 2D fitting. Finally, we contrast our approach to related work on superquadrics.

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2 NONLINEAR DEFORMABLE SUPERQUADRIC MODELS

In this section, the superquadrics and their deformations are explained and then a new modeling method is presented.

2.1 SUPERQUADRICS

Superquadrics are surfaces in 3D space that have a compact implicit representation as the set of points (x, y, z) such that

$$\left(\left(\frac{x}{a} \right)^{\frac{2}{\varepsilon_2}} + \left(\frac{y}{b} \right)^{\frac{2}{\varepsilon_2}} \right)^{\frac{\varepsilon_2}{\varepsilon_1}} + \left(\frac{z}{c} \right)^{\frac{2}{\varepsilon_1}} = 1 \qquad (1)$$

Superquadrics are topologically equivalent to sphere. They can be considered as ellipsoids with axes a, b, c whose curvature in the x, y, z directions is distorted by the influence of the exponents . (The above equation is the implicit equation for the case where the superquadric solid is in standard position with its midpoint at the origin). The exponents ε_1, ε_2 open an enormous flexibility for adjusting the shape of superquadrics in order to approximate real objects. Some basic problems in geometric modeling and object reconstruction, for example, the problem of deciding whether a point is inside or outside an object can be easily solved for superquadrics.

2.2 DEFORMATION

A globally specified deformation of a 3D solid is a mathematical function **F** which explicitly modifies the global coordinates of points in space. Mathematically, it can be represented by the equation **X** = **F**(**x**), where **x** represents the points in the undeformed solid, and **X** represents the points in the deformed solid[2]. Deformation includes tapering, bending,twisting, and so on. In this paper, we only discuss tapering deformation.

Tapering is making an object become gradually narrower toward one end of it. Mathematically, it differentially changes the length of the two global components without changing the length of the third.

To do a tapering operation along the z–axis, one should choose a tapering function depending on the z–coodinates of the points. When the tapering function f(z) = 1, the portion of the deformed object is unchanged; the object increases in size as a function of z when f′(z) > 0 and decreases in size when f′(z) < 0. The object passes through a singularity at f(z) = 0 and becomes everted when f(z) < 0.

Global tapering along the z–axis is given by the following equations:

$$\begin{cases} r = f(z) \\ X = rx \\ Y = ry \\ Z = z \end{cases} \qquad (2)$$

2.3 NONLINEAR DEFORMATION

In the previous work[2, 7, 8, 9], the f(z) function is first–order along z–axis. To increase geometric coverage of deformable models, we introduce a piecewise cubic Bezier curve as the f(z) to deform the superquadric models.

Before defining nonlinear deformable superquadric models, we first consider the piecewise cubic Bezier splines.

A Bezier curve Q(t) = (X(t),Y(t))of degree n is defined in terms of Bernstein polynomials:

$$Q(t) = \sum_{i=0}^{n} P_i B_i^n(t) \qquad (0 \leqslant t \leqslant 1) \qquad (3)$$

where the P_i are the control points, and the B_i^n are the Bernstein polynomials of degree n:

$$B_i^n(t) = C_n^i t^i (1-t)^{n-i} \quad i = 0,1,\ldots,n \qquad (4)$$

where C_n^i is the binomial coefficient $\frac{n!}{(n-i)!\ i!}$.

The most attractive feature of Bezier spline is its convex–hull property, which means that any curve defined using it smoothly follows the control points without erratic oscillations.

In practice, piecewise polynomial curves up to third order are preferred because they have second–derivate, and because they do not exhibit the oscillatory behavior of the higher orders ones. In addition, they are numerically stable and easier to compute.

Usually the piecewise splines are required to satisfy some continuity constraints such as positional, first–derivative, and geometric continuity, at the joints between successive segments. For the joint to appear smooth, we use geometric continuity which implies that the shared control point and its two neighbors need only be colinear. In this way, we can define a curve with fewer segments and do not affect the appearance of smoothness at the joint.

By interactively giving a set of control points, we can define a n–segment cubic Bezier spline:

$$\begin{cases} X_i(t) = \sum_{j=0}^{3} x_j B_j^3(t) & 1 \leqslant i \leqslant n \\ Y_i(t) = \sum_{j=0}^{3} y_j B_j^3(t) & 0 \leqslant t \leqslant 1 \end{cases} \qquad (5)$$

where,

$$\begin{cases} B_0^3(t) = (1-t)^3 \\ B_1^3(t) = 3t(1-t)^2 \\ B_2^3(t) = 3t^2(1-t) \\ B_3^3(t) = t^3 \end{cases}$$

Thus, let us consider the NDS models.

Suppose we have a piecewise cubic Bezier spline Q(t) = (x(t), z(t)), which we want to deform a superquadrics. Let $x(t) \in [0, 1]$, $z(t) \in [-c, c]$ and $Q(t)$ is monotonous for any given z. The deformation is expressed by:

$$r = f(z) = x(t) = x(z^{-1}(t))$$

For any 3D point (x, y, z) on the superquadric surface, we can use $z = z(t)$ to find the correspond segment number $j \in [1, n]$ and $t \in [0, 1]$ by a cubic root finding method. From these results, we can compute $r = x(t)$. Finally point (X, Y, Z) on the deformed superquadric model can be found by equations(2).

By moving the control points of Bezier spline, the model can deformed interactively as if it was made of a flexible material. Figure 1 shows an example of NDS models. The bottle and cup are modeled by NDS and graphically generated by ray tracing technique.

Figure 1. Objects represented by NDS models

3 IMAGE RECONSTRUCTION OF NDS MODELS

In order to demonstrate the effectiveness of our model, we developed a framework for 3D object reconstruction from single monocular image.

For simplicity, we assume that the objects in the image are straight axial symmetry and observed under orthographic projection. The reconstruction procedure is as following:

1. The object silhouette is found by an edge detector and contour tracking;
2. Compute the orientation and lenghth of long axis (for z axis and c in Eq.(1)) and short axis (for x, y axes and (a, b) in Eq.(1)) using moment analysis;
3. A piecewise cubic Bezier curve automatically fits to the contour of the object;
4. Using the spline to deform the superquadrics (we initialized the model to a cylindrical shape by setting $\varepsilon_1 = 0.01$, and $\varepsilon_2 = 2$), we can finally obtain the reconstructed object.

We have built a system to demonstrate our approach to models and reconstruction of 3D objects. The system has been implemented in C on a SUN 3 workstation. One example of reconstruction process is illustrated in Fig.2. Fig.2(a) shows the extracted closed contour. The results of spline fitting are presented in Fig.2(c). From the control points, we can find that spline is represented by three segments of Bezier curves. The last figure shows a graphically rendered view of the reconstructed model.

4 DISCUSSION

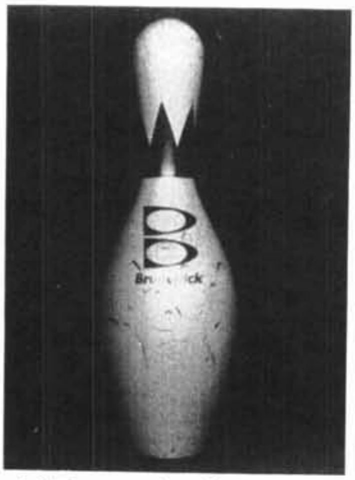
A new method of representation and reconstruction of 3D objects has been presented. It uses a piecewise cubic Bezier spline to deform the superquadrics. We consider the NDS model inspired by the linear deformed superquadrics, generalized cylinder and spline models. However, it distinguishes from them in some aspects.

In the famous ACRONYM system[4], objects are represented as constructions of generalized cylinders. Although they are volumetric models, they do not have a inside–outside function which facilitate the reconstruction and recongnition processes. The NDS model preserves this feature of superquadrics.

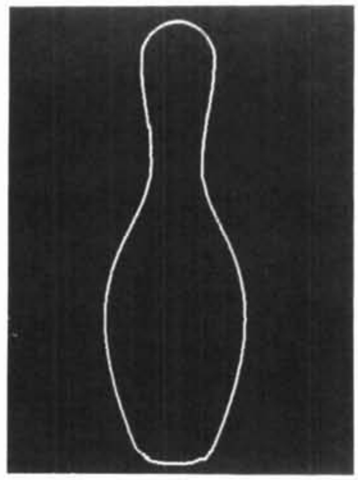
Pentland[7] first applied superquadrics and deformation to primitive modeling for object recognition. A number of subsequent work have achieved considerable success in 3D object representation and recovery[6, 8, 9]. But they are still difficult to model the curved objects, such as bottles and cups. Our method broads representational power of deformable models by just adds 3n+1 (n is the segment number) control points. Spline models[3] can reconstruct surface very detail, however, NDS models need fewer data to model the volumetric objects.

In conclusion, the NDS models have the global degrees of freedom of superquadrics to capture the salient features of shape that are appropriate for matching against object prototypes, whereas it also have the local degrees of freedom of nonlinear deformation which allow the reconstruction of fine scale struture and the natrual irregularities of real world data. NDS models are therefore suitable for use in both visual reconstruction and recognition tasks. The results of the NDS model reconstruction in our implemented agorithm have confirmed this.

To recognize more complicated parts, such as NDS model adds bending deformation, from range image is our future research direction.

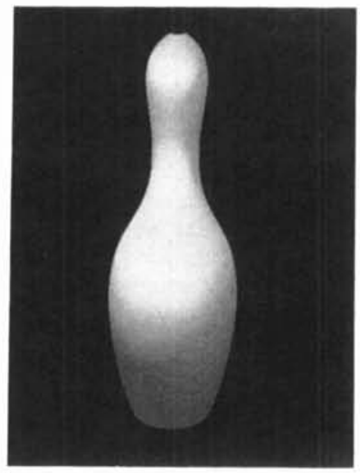


(a) The grey–level image



(b) The contour of the bowling

(c) The fitted piecewise cubic Bezier curve and its control points



(d) Reconstructed model

Figure 2. Reconstruction of a bowling

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