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DETERMINING SURFACE ORIENTATION BY UNI-DIRECTIONAL PHOTOMETRIC FLOW FIELDS

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ABSTRACT

In the Shape from Shading methods, an extended approach named Uni-directional Photometric Flow Fields is proposed in this paper. The reflectance property of the surface material is the uniform perfect diffuser, and the light source produces a uniform parallel beam and the illuminating direction is varied slightly only in the azimuth direction. The local surface orientation can be determined from the image density and its directional derivatives. It is shown that the zenith angle of the illuminating direction is also determined analytically. The errors caused by calculating the directional derivatives using the numerical differences from three input image densities are evaluated, and the effectiveness of the shape reconstruction algorithm is demonstrated by computer simulations.

1. INTRODUCTION

The *Shape from Shading* formulated by Horn[1],[2] is one of the useful techniques that reconstruct the shape of the three-dimesional object from its two-dimensional shading image. And then, *Photometric Stereo* is introduced by Woodham[3] to determine the local surface orientation of the object from three input images under the independent parallel light beams illumination. In comparison with *Photometric Stereo* that requires three large variations of the illuminating directions, *Photometric Flow Fields* (PFF) that utilizes the illuminating directions varied in the small ranges is proposed by Wolff[4]. The local surface orientaion is determined from the image density and its directional derivatives obtained by varying the illuminating direction. In PFF, two degrees of freedom of the illuminating direction are necessary, one of which is varied in the azimuth direction and another of which is varied in the zenith direction. More simple method named *Uni-directional Photometric Flow Fields* (UPFF) is proposed in this paper, in which the illuminating direction is varied slightly only in the azimuth direction.

Under the condition that the object surface is the uniform perfect diffuser, it is shown that the local surface orientation can be obtained at any surface element from the image density and its directional derivatives (i.e., the first order and the second order partial directional derivatives of the image density with respect to the azimuth angle) under the known illuminating direction. Furthermore, in case that the zenith angle of the illuminating is unknown, the angle can be determined from the information of more than two surface elements.

In the algorithm for determining the surface orientation, the approximation by the numerical difference method is adopted in calculating the directional derivatives from measured image densities. Computer simulations show the evaluation of errors in calculating those directional derivatives for the various values in the small changes of the azimuth angle of the illuminating direction, and show the effectiveness of this algorithm by demonstrating the shape reconstruction for the curved objects.

2. ILLUMINATING EQUATION

2.1 Coordinate System

Figure 1 shows the coordinate system of UPFF. The observation system is the orthographic projection, and the light source produce a uniform parallel beam. Under these conditions, three vectors at a surface element are defined as follows. Unit vector **n** of the surface element is given by

$$\mathbf{n} = \frac{(-p, -q, 1)}{\sqrt{p^2+q^2+1}} \tag{1}$$

where p and q are the first order partial derivatives of z with respect to x and y, i.e., they are the gradient components of a surface elements.

$$p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y} \tag{2}$$

Unit vector **s** is the light source direction vector defined as follows, and it is constant at all of the surface elements.

$$\mathbf{s} = (\sin\alpha\cos\beta,\ \sin\alpha\sin\beta,\ \cos\alpha) \tag{3}$$

where α and β are the zenith angle and azimuth angle of illuminating direction, respectively. Unit vector **v** is the observation direction vector, and it is also constant at all of the surface elements.

$$\mathbf{v} = (0, 0, 1) \tag{4}$$

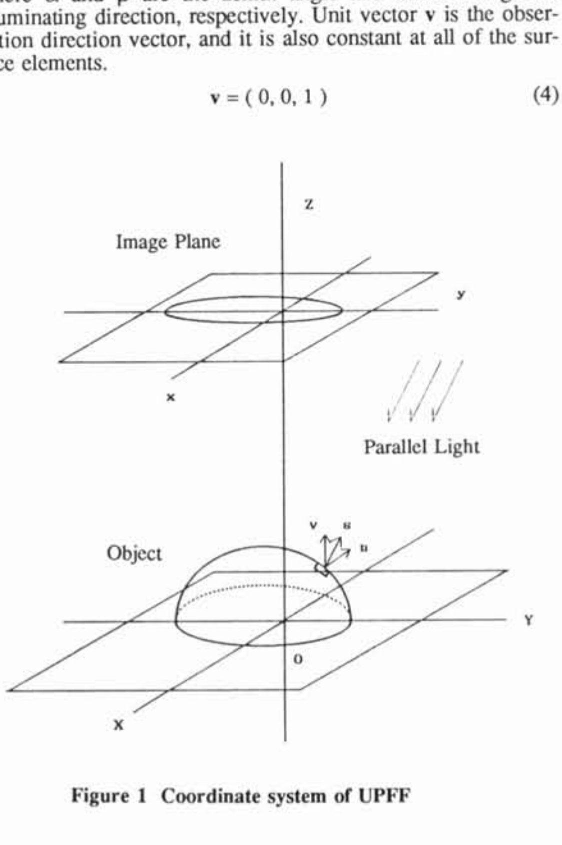

Figure 1 Coordinate system of UPFF

2.2 Illuminating Equation

For an object with perfectly diffused surface, the image density D is determined by the following illuminating equation.

$$D = C\,(\mathbf{n}\cdot\mathbf{s}) = C\frac{-p\sin\alpha\cos\beta - q\sin\alpha\sin\beta + \cos\alpha}{\sqrt{p^2+q^2+1}} \tag{5}$$

where C is a proportional constant determined by the luminous intensty of the light sourece, the diaphragm of a camera lens, and the surface albedo under the assumption that Gamma property is linear.

3. PRINCIPLE OF UPFF

3.1 Determination of Local Surface Orientation

Let us consider the directional derivatives of the image density as a function described in the illuminating equation. In PFF, the illuminating direction is varied slightly both in the zenith direction and in the azimuth direction. While in UPFF, the illuminating direction is varied slightly only in the azimuth direction, as illustrated in Figure 2.

The directional derivatives of image density are derived as follows:

$$\partial D_b = C\frac{p\sin\alpha\sin\beta - q\sin\alpha\cos\beta}{\sqrt{p^2+q^2+1}} \tag{6}$$

$$\partial D_{bb} = C\frac{p\sin\alpha\cos\beta + q\sin\alpha\sin\beta}{\sqrt{p^2+q^2+1}} \tag{7}$$

where

$$\partial D_b = \frac{\partial D}{\partial\beta},\quad \partial D_{bb} = \frac{\partial^2 D}{\partial\beta^2}$$

for abbreviation. Operating Eq.(6)/Eq.(5) and Eq.(7)/Eq.(5), and then

$$\frac{\partial D_b}{D} = \frac{p\sin\alpha\sin\beta - q\sin\alpha\cos\beta}{-p\sin\alpha\cos\beta - q\sin\alpha\sin\beta + \cos\alpha} \tag{8}$$

$$\frac{\partial D_{bb}}{D} = \frac{p\sin\alpha\cos\beta + q\sin\alpha\sin\beta}{-p\sin\alpha\cos\beta - q\sin\alpha\sin\beta + \cos\alpha} \tag{9}$$

are obtained. Equations (8) and (9) are the linear equations of p and q. By solving the equations (8) and (9) with respect to p and q, the next relations are obtained.

$$p = \frac{\partial D_b\sin\beta + \partial D_{bb}\cos\beta}{(D+\partial D_{bb})\tan\alpha} \tag{10}$$

$$q = \frac{-\partial D_b\cos\beta + \partial D_{bb}\sin\beta}{(D+\partial D_{bb})\tan\alpha} \tag{11}$$

Therefore, the local surface orientation of an object is derived analytically from image density and its directional derivatives corresponding to the surface element under the known illuminating direction.

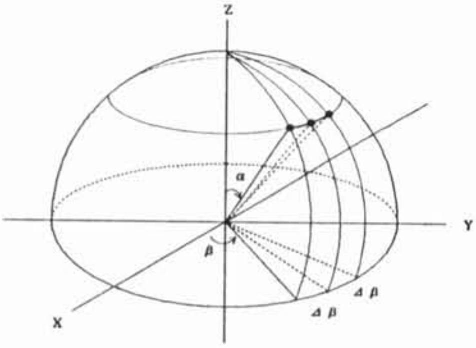

Figure 2 Illuminating Directions

3.2 Estimation of α for Unknown Surface Shape

Substituting equations (10) and (11) into illuminating equation (5) gives

$$\partial D_b^2 + \partial D_{bb}^2\tan^2\alpha - C^2\sin^2\alpha = 0 \tag{12}$$

Replacing $u = \sin^2\alpha$ and arranging the equation (12) respect to variable u,

$$C^2u^2 - (C^2 - D^2 + \partial D_b{}^2 - 2D\,\partial D_{bb})u + \partial D_b^2 + \partial D_{bb}^2 = 0 \tag{13}$$

is deduced. Since equation (13) is a quadratic equation of u and generally there exists two solutions, the value of u can not be determined uniquely. To get the unique solution, the additional information of another surface element is introduced. From the condition that the light source produces parallel beam, the light source direction is constant at all of the surface elements. Suppose that D_m and D_n are the image densities corresponding to the different surface elements m and n, then the unit light source direction vector $\mathbf{s}_m$ and $\mathbf{s}_n$ are the same. Therefore, the following simultaneous quadratic equations of u are obtained, and have the common solution.

$$C^2u^2 - (C^2 - D_m^2 + \partial D_{b_m}{}^2 - 2D_m\partial D_{bb_m})u + \partial D_{b_m}{}^2 + \partial D_{bb_m}{}^2 = 0 \tag{14}$$

$$C^2u^2 - (C^2 - D_n^2 + \partial D_{b_n}{}^2 - 2D_n\partial D_{bb_n})u + \partial D_{b_n}{}^2 + \partial D_{bb_n}{}^2 = 0 \tag{15}$$

From equations (14) and (15), the solution is given by

$$u = \frac{\partial D_{b_m}{}^2 + \partial D_{bb_m}{}^2 - \partial D_{b_n}{}^2 - \partial D_{bb_n}{}^2}{\partial D_m{}^2 - \partial D_{b_m}{}^2 + 2D_m\partial D_{bb_m} - \partial D_n{}^2 + \partial D_{b_n}{}^2 - 2D_n\partial D_{bb_n}} \tag{16}$$

Consequently, the zenith angle of the illuminating direction is uniquely determined as follows.

$$\alpha = \sin^{-1}\sqrt{u} \tag{17}$$

4. COMPUTER SIMULATIONS

4.1 Approximation of Directional Derivatives

In UPFF, the directional derivatives ∂Db and ∂Dbb have to be calculated by the numerical difference approximation for all surface elements. For the approximation, finite difference method such as forward difference and central difference are considered. In these notation, $\partial Db*$ and $\partial Dbb*$ mean the approximated values of ∂Db and ∂Dbb respectively.

< forward difference approximation >

$$\partial D_b{}^* = \frac{D_b - D}{\Delta\beta},\quad \partial D_{bb}{}^* = \frac{D_{bb} - 2D_b + D}{(\Delta\beta)^2} \tag{18}$$

< central difference approximation >

$$\partial D_b{}^* = \frac{D_b - D_{-b}}{2\Delta\beta},\quad \partial D_{bb}{}^* = \frac{D_b - 2D + D_{-b}}{(\Delta\beta)^2} \tag{19}$$

where Db, D_{-b} and Dbb are the image densities measured in the azimuth direction $\beta+\Delta\beta$, $\beta-\Delta\beta$ and $\beta+2\Delta\beta$ respectively. In the following notation, the illuminating equation (5) is represented as $D = C\Phi(\alpha, \beta)$ then Db, D_{-b} and Dbb are defined as follows.

$$\begin{aligned} D_b &= C\Phi(\alpha, \beta+\Delta\beta) \\ D_{-b} &= C\Phi(\alpha, \beta-\Delta\beta) \\ D_{bb} &= C\Phi(\alpha, \beta+2\Delta\beta) \end{aligned} \tag{20}$$

Therefore, three image densities D, Db and Dbb are required in the forward difference method, while D, D_{-b} and Db are required in the central difference method.

4.2 Evaluation of Errors by Approximation

The effects of errors by approximation are to be inspected before reviewing the theory of shape reconstruction. In the simulation, sixty-four surface elements are selected at the equal intervals on the corresponding image plane from the hemisphere object whose surface orientations have every combination of directions. Some kinds of the mean square errors are evaluated. The result is shown in Figure 3. The mean square error of ∂D_b is defined as follows and shown as curve (a) and (c) in Figure 3,

$$e_1 = \frac{1}{N}\sum_{i=1}^{N}(\partial D_b - \partial D_b^{*})^2 \tag{21}$$

and that of ∂D_{bb} is defined as follows and shown as curve (b) and (d) in Figure 3,

$$e_2 = \frac{1}{N}\sum_{i=1}^{N}(\partial D_{bb} - \partial D_{bb}^{*})^2 \tag{22}$$

The evaluation by the true values of (p, q) are shown as (a) and (b), and the evaluations by the estimated values of (p, q) calculated from equation (12) and (13) as shown as (c) and (d) in Figure 3.

The mean square error of the reconstructed normal vector is evaluated and shown as curve (e) in Figure 3,

$$e_3 = \frac{1}{N}\sum_{i=1}^{N}B \tag{23}$$

where B is as follows,

$$B = \left[\frac{p}{\sqrt{p^2+q^2+1}} - \frac{p_e}{\sqrt{p_e^2+q_e^2+1}}\right]^2 + \left[\frac{q}{\sqrt{p^2+q^2+1}} - \frac{q_e}{\sqrt{p_e^2+q_e^2+1}}\right]^2 + \left[\frac{1}{\sqrt{p^2+q^2+1}} - \frac{1}{\sqrt{p_e^2+q_e^2+1}}\right]^2$$

where p_e and q_e are the estimated p and q by equation (10) and (11) respectively.

The mean square error of the estimated zenith angle of illuminating directions is evaluated and shown as curve (f) in Figure 3.

$$e_4 = \frac{1}{N}\sum_{i=1}^{N}(\alpha - \alpha_e)^2 \tag{24}$$

where α_e is the estimated α by equation (17).

From this simulation, it is apparent that the result by the central difference approximation method is better than that by the forward differece method. In the central difference approximation, the errors increase with $\Delta\beta$ decrease less than 0.01[°] of $\Delta\beta$. Generally in the difference approximation, the rounding error appears at too small variation. In the central difference method, it is considered that the effect of the rounding error appears at larger value of $\Delta\beta$ than that of the forward difference approximation.

4.3 Shape Reconstruction Algorithm

The algorithm for shape reconstruction is shown in this section. UPFF reconstruct the surface orientation of the curved objects according to the following steps by the central difference approximation.

[*Step1*] The values of the proportional constant C is set to be the maximum intensity value of all of the image pixels. The azimuth angle β and its angle difference $\Delta\beta$ of the illuminating direction are set as known constants.

[*Step2*] Measure three image densities D, D_{-b} and D_b of curved object, and calculate the directional derivatives ∂D_b and ∂D_{bb} using equation (19) for all of the surface elements.

[*Step3*] Estimate the zenith angle α of the illuminating direction if it is unknown by using equation (17) from the information of two arbitrary different surface elements.

[*Step4*] Determine surface orientation (p, q) for all the curved surface using equations (10) and (11).

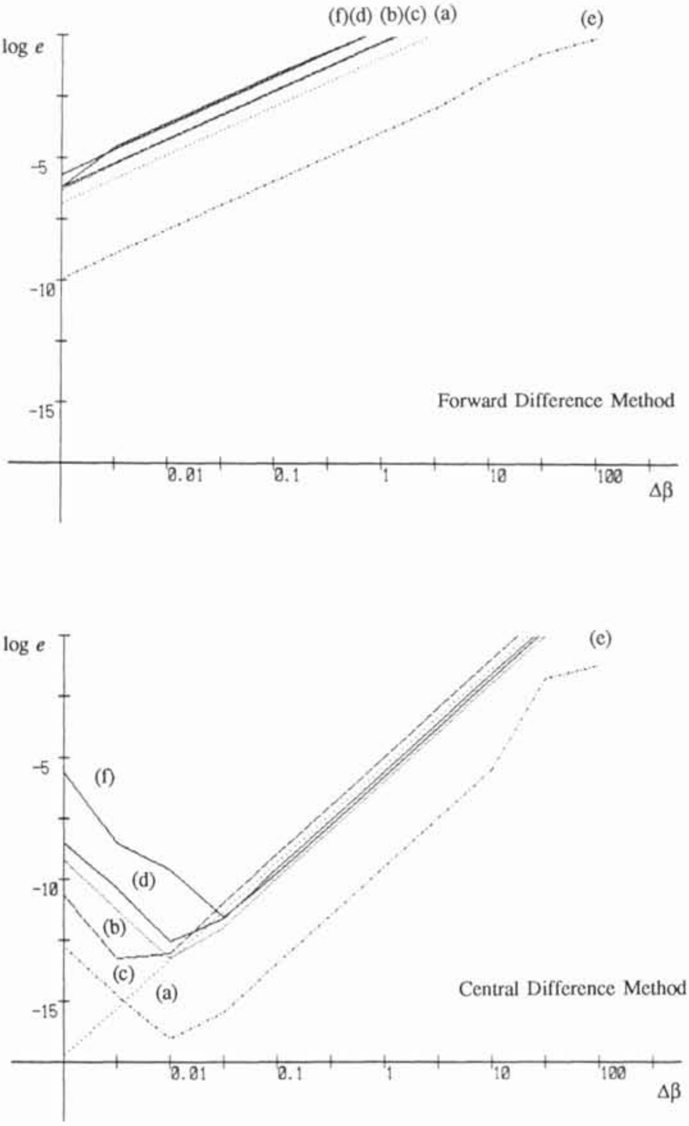

Figure 3 Evaluation of Errors by Approximatioin
(Object: Hemisphere Lighting: α=30° β=45°)
(a) *e1*: evaluated by true (p, q)
(b) *e2*: evaluated by true (p, q)
(c) *e1*: evaluated by estimated (p, q)
(d) *e2*: evaluated by estimated (p, q)
(e) *e3*: (p, q) estimation error
(f) *e4*: α estimation error

4.4 Surface Orientation for Curved Surfaces

Setting the original α to be 30[°], β to be 45[°], and $\Delta\beta$ to be 0.01[°], UPFF is demonstrated by the computer simulations. Simulated objects are a hemisphere, a semi-paraboloid and a semi-ellipsoid. Figure 4 shows input images of these objects. The simulated results are shown in Figure 5.1--5.3. In each figure, the needle map (a) shows the true orientation of the curved surface, and the map (b) shows the reconstructed orientation. The order of the difference between the estimated α and the original α was about 2.37×10^{-10}.

From the reason that there are many surface elements commonly illuminated by three slightly varied directions in UPFF, there is an advantage that the number of the reconstructed surface elements by UPFF is more than that by *Photometric Stereo*.

5. CONCLUSION

A new photometric method named UPFF is proposed and discussed in this paper. The method adopts the principle of the monocular vision and the parallel light beam illumination.

Conditions used in this paper are that the object is with the perfectly diffused surface and that input image densities are the ideal analog quantities.

Under these conditions, an algorithm is developed to determine the surface orientation of the curved object including the case that the zenith angle of the illuminating direction is unknown. In comparison with PFF, UPFF can utilize more simple mechanism that illuminating direction is varied only in the azimuth direction.

It should be noted that this paper describes the theoretical aspect of UPFF and does not cover the practical applications. Those practical problem should be studied by the development on the hardware of the image processing unit.

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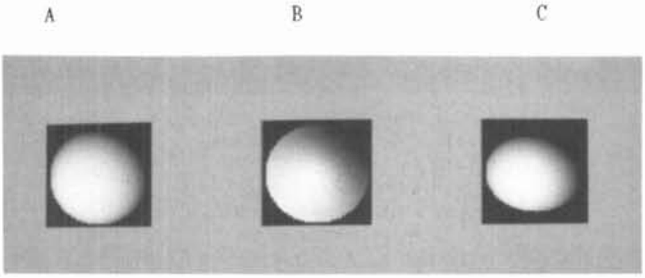

Figure 4 Input Images of Curved Objects
(A: Hemisphere B: Semi-paraboloid C: Semi-ellipsoid)

(a) Needle Map by True (p, q) (b) Needle Map by Estimated (p, q)

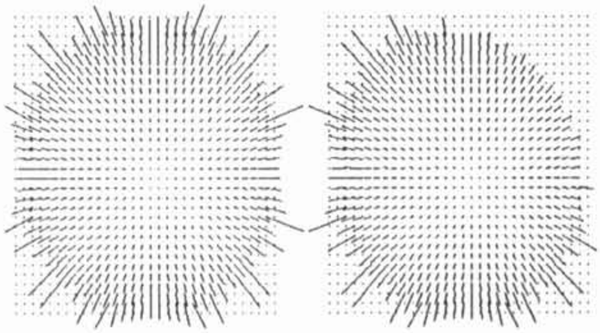

Figure 5-1 Reconstructed Shape of Curved Object
(Object: Hemisphere Lighting: α=30° β=45° $\Delta\beta$=0.01°)

(a) Needle Map by True (p, q) (b) Needle Map by Estimated (p, q)

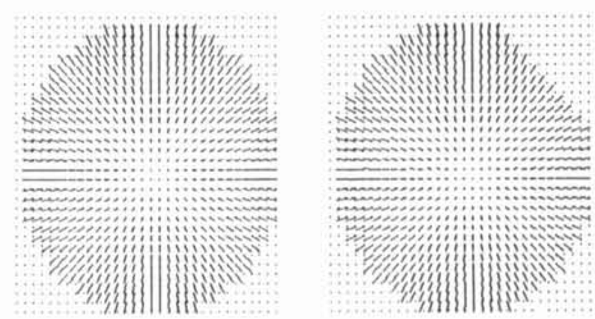

Figure 5-2 Reconstructed Shape of Curved Object
(Object: Semi-paraboloid Lighting: α=30° β=45° $\Delta\beta$=0.01°)

(a) Needle Map by True (p, q) (b) Needle Map by Estimated (p, q)

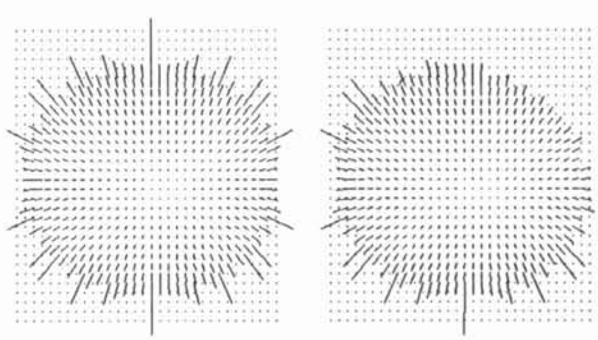

Figure 5-3 Reconstructed Shape of Curved Object
(Object: Semi-ellipsoid Lighting: α=30° β=45° $\Delta\beta$=0.01°)