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# QUANTITATIVE SHAPE RECOVERY OF AN OBJECT FROM A SINGLE VIEW

by

Xiang-Shu Wei, Qing-Yun Shi and Min-De Cheng

Information Science Center of Peking
University, Beijing, China

Abstract — A method for quantitative shape recovery of an object from a single picture is proposed in this paper. By introducing a 3-dimensional dual space, four basic constraints between a polyhedron and its central projection image have been found. A procedure is then given to recursively obtain the quantitative descriptions of planar-surfaces of the polyhedron. A general solution form is also established to represent the polyhedrons which are possible to generate the given projection image. How to obtain a natural solution from the general solution form by the use of gray level information is finally discussed.

## I. Introduction

Stereoscopic shape recovery has been investigated quite extensively, but there is not much work on quantitative shape recovery from a single view. The difficult associated with this task is that during the process of image formation, some spatial information of an object has been lost. In order to recover the 3-dimensional shape of the object, we must utilize enough constraints between the object and its projection image.

For the case of orthographic projection, Mackworth [1] discovered that the gradients of two planes of a polyhedron in gradient space should be on a line which is perpendicular to the projection line of the common edge of those two planes. Using this constraints, the line-labeling task can be completed for a line drawing. Kanade [2] combined this constraint with surface cues such as parallelism and skew symmetry to perform quantitative shape recovery in gradient space. The gradients of visible planar-surfaces of a polyhedron corresponding to a given line drawing have been obtained. However, the constraints infered in gradient space give only necessary conditions for a line drawing correctly representing a polyhedron, so there is no assurance that the recovered shape forms a polyhedral scene (see [3] and [4] for example).

In this paper, we introduce a 3-dimensional dual space instead of the gradient space, and find four basic constraints between a polyhedron and its central projection image. Based on these constraints, a procedure can be constructed for recursively obtaining the quantitative descriptions of planar-surfaces of the polyhedron. A general solution form is also established to represent the polyhedrons which are possible to generate the given projection image. Finally, we discuss how to get a natural solution from the general solution form by the use of gray level information.

## II. The dual space

Let D be a line drawing representing the projection image of a polyhedron in 3D Euclidean space $R^3$. A node $\mathbf{v}_i$ on D corresponds to a vertex $V_i$ on the polyhdron. A line $l_i$ on D corresponds to an edge $L_i$ on the polyhedron. And a region $s_i$ on D corresponds to a planar-surface $S_i$ on the polyhedron. In what follows, we adopt the so-called non-accidental assumption, so that line drawing as shown in Fig. 1 is out of our consideration.

Given a line drawing D, we take the image plane as xy-plane. Let O be the origin on the plane, and z-axis point to the view direction. The view point is $O'=(0,0,f)$, where $f>0$ is a camera parameter. Fig. 2 shows the process of image formation.

Suppose that $l_i$ is a line on D, let $[l_i]$ denote the projection plane which generates $l_i$. If on xy-plane the equation for $l_i$ is $p_ix+q_iy+f=0$, then in $R^3$ the equation for $[l_i]$ is $p_ix+q_iy-z+f=0$ due to $O'=(0,0,f)$ and $l_i$ being on $[l_i]$. In this way, we can obtain the equations of all projection planes which generate the line drawing D.

A visible plane $S_i$ on the polyhedron corresponding to D can be represented by a normal form equation $p_ix+q_iy+d_iz+f=0$, where $p_i$, $q_i$, and $d_i$ are parameters. All parameter vectors of the form $(p_i,q_i,d_i)$ make up a new 3-dimensional space, which is called the dual space and is denoted by D(f). Since a plane $S_i$ in $R^3$ corresponds to a point $(p_i,q_i,d_i)$ in D(f), so the latter is sometimes called $(p_i,q_i,d_i)$-plane. As mentioned above, every projection plane has the form $[l_i]=(p_i,q_i,-1)$. Hence, it lies on the plane $d=-1$ in D(f). Let L be the set of projection planes generating the line drawing D, then L is a known set of points in D(f). On the other hand, the set of visible planes on the corresponding polyhedron is an unknown set of points S in D(f). It is easy to see that the quantitative shape recovery of an object corresponding to the line drawing D can be stated as the following problem in D(f): to find out the unknown set of points S from the known set of points L. In order to solve this problem, we need to descover some constraints between S and L, which is the major task of Section III.

## III. Basic constraints

Suppose that $s_i$ is a region on the line drawing D corresponding to a visible plane $S_i$ of the polyhedrons. Let $l_j$ be a boundary line of $s_i$ and $V_{j1}=(x_1,y_1)$, $V_{j2}=(x_2,y_2)$ be the end points of $l_j$ (see Fig. 2). We can construct four planes in the dual space D(f) as follows:

$\pi_1 : x_1p+y_1q+f=0,$

$\pi_2 : x_2p+y_2q+f=0,$

$\pi_3 : x_1p+y_1q-fd=0,$

$\pi_4 : x_2p+y_2q-fd=0.$

Let $I(l_j)$ be the region bounded by these four planes and containing the positive part of d-axis. Obviously, $(p,q,d)\in I(l_j)$ in space D(f) if and only if

$(x_1p+y_1q+f)(x_1p+y_1q-fd)\leq 0$

$(x_2p+y_2q+f)(x_2p+y_2q-fd)\leq 0$

Thus, we have

Lemma 1. If $l_j$ is a boundary line of region $s_i$, and $s_i$ is the projection image of plane $S_i$ in $R^3$, then $S_i=(p_i,q_i,d_i)\in I(l_j)$ in D(f).

Now let $N_i=\{l_j | l_j$ is a boundary line of region $s_i\}$, then a region

$$C(s_i) = \bigcap_{l_j\in N_i} I(l_j)$$

can be obtained in space D(f). Since every $I(l_j)$ contains the positive part of d-axis, so $C(s_i)$ is not empty. From Lemma 1, we obtain:

Constraint 1. If region $s_i$ is the projection image of plane $S_i$ in $R^3$, then $S_i\in C(s_i)$ in the dual space D(f).

Constraint 1 means that the shape and location of a region $s_i$ on the line drawing D restricts the corresponding plane $S_i$. Not every plane can generate the projection image like $s_i$. There exist other constraints between $S_i\in S$ and $[l_k]\in L$, which are given in the following.

Constraint 2. If two planes $S_i$ and $S_j$ in $R^3$ have a common edge $L_k$, and the projection image of $L_k$ is $l_k$ (see Fig. 3), then the points $S_i$, $S_j$ and $[l_k]$ in space D(f) are colinear.

Constraint 3. If $L_{k1}$ is an edge of plane $S_i$, $L_{k2}$ is an edge of plane $S_j$, and $L_{k1}$, $L_{k2}$ are coplanar in $R^3$ (see Fig. 4), then the points $S_i$, $S_j$, $[l_{k1}]$ and $[l_{k2}]$ are coplanar in space D(f).

If a plane $S_j$ in $R^3$ is occluded by a plane $S_i$ (as shown in Fig. 5), then the common boundary line $l_k$ of the corresponding region $s_j$ and $s_i$ is called an occlusion line. Let the end points of $l_k$ be $v_{k1}=(x_1,y_1)$ and $v_{k2}=(x_2,y_2)$. Make two lines $l'$: $x_1p+y_1q+f=0$ and $l''$: $x_2p+y_2q+f=0$ on the plane d=-1 in space D(f). Suppose that $S_i=(p_i,q_i,d_i)$, then make two planes $\pi'$ and $\pi''$ in space D(f) such that $\pi'$: $x'p+y'q+z'd+f=0$ passes through $S_i$ and the line $l'$, and $\pi''$: $x''p+y''q+z''d+f=0$ passes through $S_i$ and the line $l''$. Let $J(S_i,l_k)$ denote the open region bounded by planes $\pi'$, $\pi''$ and d=-1, and containing the origin. A point $(p,q,d)\in J(S_i,l_k)$ in D(f) if and only if $(x'p+y'q+z'd+f)(d+1)>0$ and $(x''p+y''q+z''d+f)(d+1)>0$. Thus we have

Lemma 2. If a plane $S_j$ in $R^3$ is occluded by a plane $S_i$, and the occlusion line is $l_k$ as shown in Fig. 5, then $S_j\in J(S_i,l_k)$ in space D(f).

Furthermore, if $V_{k1}$ (or $V_{k2}$) is on the plane $S_j$ in $R^3$, then $S_j$ is on the plane $\pi'$ (or $\pi''$) in D(f) (see Fig. 5). Especially, when both $V_{k1}$ and $V_{k2}$ are on the plane $S_j$ in $R^3$, then $S_j$ must be on the intersection line of $\pi'$ and $\pi''$ in D(f). In this case, planes $S_i$ and $S_j$ are intersected at line $L_k$, and the projection image $l_k$ of $L_k$ is no longer an occlusion line. Hence, for generality, let $\bar{J}(S_i,l_k)$ be the closed region consisting of $J(S_i,l_k)$ and its boundary. Then combining Lemma 1 and Lemma 2, we obtain:

Constraint 4. Let the projection image of an edge $L_k$ of plane $S_i$ be $l_k$. If $l_k$ is the common boundary line of regions $s_i$ and $s_j$ on the line drawing D (see Fig. 5), then the corresponding $S_j\in I(l_k)\cap\bar{J}(S_i,l_k)$ in space D(f).

Since region $I(l_k)$ contains the positive part of d-axis, and $\bar{J}(S_i,l_k)$ contains the origin, so that $I(l_k)\cap\bar{J}(S_i,l_k)$ is not empty. In what follows, we use $E(S_i,l_k)$ to represent it.

Up to now, we have discovered four constraints which reflect the relationship between the known set of points L and the unknown set of points S in space D(f). One can refer to [6] for the proofs of their correctness. Using these constraints, we can construct a procedure to obtain the elements of S, which quantitatively describe the 3-dimensional shape of an object corresponding to the given line drawing.

## IV. The Procedure

In this section, we utilize four basic

constraints given above to construct a procedure for the solution of unknown point set S in space D(f).

The region $C(S_i)$ given by Constraint 1 is used to determine the initial conditions. Constraints 2 and 3 are used to establish the following solution rules.

Rule 1. If $S_k \in S$, and in $R^3$ the planes $S_k$ and $S_i$ have a common edge $L_{k1}$; the planes $S_k$ and $S_j$ have a common edge $L_{k2}$, then in space D(f),

$$S_k = P_1(S_i, [l_{k1}], S_j, [l_{k2}]),$$

where $P_1$ is a linear operator which makes one line passing through the points $S_i$ and $[l_{k1}]$, another line passing through the points $S_j$ and $[l_{k2}]$, and then takes the intersection point of them.

Rule 2. If $S_k \in S$, and in $R^3$ the planes $S_k$ and $S_i$ have a common edge $L_{k1}$, besides, the edge $L_{k2}$ of plane $S_k$ and the edge $L_{k3}$ of plane $S_j$ are coplanar, then in space D(f),

$$S_k = P_2(S_i, [l_{k1}], S_j, [l_{k2}], [l_{k3}]),$$

where $P_2$ is a linear operator which makes a line passing through the points $S_i$ and $[l_{k1}]$, and a plane passing through the points $S_j$, $[l_{k2}]$ and $[l_{k3}]$, and then takes the intersection point of them.

Rule 3. If $S_k \in S$, and in $R^3$ the plane $S_k$ has three edges $L_{k1}$, $L_{k2}$ and $L_{k3}$, in which $L_{k1}$ and the edge $L_{k4}$ of plane $S_{i1}$ are coplanar; $L_{k2}$ and the edge $L_{k5}$ of plane $S_{i2}$ are coplanar; $L_{k3}$ and the edge $L_{k6}$ of plane $S_{i3}$ are coplanar, then in space D(f),

$$S_k = P_3(S_{i1}, [l_{k1}], [l_{k4}], S_{i2}, [l_{k2}], [l_{k5}], S_{i3}, [l_{k3}], [l_{k6}]),$$

where $P_3$ is a linear operator which makes one plane passing through the points $S_{i1}$, $[l_{k1}]$ and $[l_{k4}]$, one plane passing through the points $S_{i2}$, $[l_{k2}]$ and $[l_{k5}]$, and the other plane passing through the points $S_{i3}$, $[l_{k3}]$ and $[l_{k6}]$, and then taking the intersection of them.

Constraint 4 will be used to determine the spatial relations between the planes obtained.

Now, we are going to construct the procedure. To make the statements more clear, we explain some notations first.

r is a ternary relation. $(s_i, l_k, s_j) \in r$ if and only if regions $s_i$ and $s_j$ have a common boundary line $l_k$ on the line drawing D.

$r_1$ is a subset of r. $(s_i, l_k, s_j) \in r_1$ if and only if it belongs to r and two end points of $l_k$ are not the "T" type nodes on the line drawing D.

$r_2$ is a subset of r. $(s_i, l_k, s_j) \in r_2$ if and only if it belongs to r, and $l_k$ has an end point of "T" type with $s_i$ on the upper side (see Fig. 5).

$R_1$ is a ternary relation. $(S_i, L_k, S_j) \in R_1$ if and only if planes $S_i$ and $S_j$ are intersected at the edge $L_k$.

$R_2$ is a ternary relation. $(S_i, L_k, S_j) \in R_2$ if and only if plane $S_i$ occludes $S_j$ at the edge $L_k$.

Obviously, the ternary relations r, $r_1$ and $r_2$ can be obtained directly from the given line drawing, but $R_1$ and $R_2$ can only be obtained by the execution of quantitative shape recovery procedure. The procedure is as follows:

Procedure 1. Quantitative shape recovery of an object corresponding to a given line drawing D.

Step 1. Set $S=\phi$, $R_1=\phi$ and $R_2=\phi$.
Select $S_i=(p_i, q_i, d_i)$ in space D(f) such that $(p_i, q_i, d_i) \in C(s_i)$, and let $S=S \cup \{S_i\}$.
If $(s_i, l_j, s_k) \in r_1$, take a point $(p_k, q_k, d_k)$ on the line passing through the points $S_i$ and $[l_j]$ in D(f), and such that $(p_k, q_k, d_k) \in C(s_k)$.
Then let $S_k=(p_k, q_k, d_k)$ and go to step 3.

Step 2. If $S_i, S_j \in S$, and $(s_k, l_{k1}, s_i)$, $(s_k, l_{k2}, s_j) \in r_1$, then let $S_k=P_1(S_i, [l_{k1}], S_j, [l_{k2}])$ and go to step 3.
If $S_i, S_j \in S$, and $(s_k, l_{k1}, s_i)$, $(s_k, l_{k2}, s_t)$, $(s_t, l_{k3}, s_j) \in r_1$, then let $S_k=P_2(S_i, [l_{k1}], S_j, [l_{k2}], [l_{k3}])$ and go to step 3.
If $S_{i1}, S_{i2}, S_{i3} \in S$, and $(s_k, l_{k1}, s_{t1})$, $(s_{t1}, l_{k2}, s_{i1})$, $(s_k, l_{k3}, s_{t2})$, $(s_{t2}, l_{k4}, s_{i2})$, $(s_k, l_{k5}, s_{t3})$, $(s_{t3}, l_{k6}, s_{i3}) \in r_1$, then let $S_k=P_3(S_{i1}, [l_{k1}], [l_{k4}], S_{i2}, [l_{k2}], [l_{k5}], S_{i3}, [l_{k3}], [l_{k6}])$ and go to setep 3.
Otherwise, stop.

Step 3. Let $S_k$ be the plane currently considered. Take $(s_k, l_j, s_i) \in r$ and $S_i \in S$.
If $S_k, [l_j], S_i$ are colinear in space D(f), then let $(S_k, L_j, S_i) \in R_1$ and go to step 4.
If $S_k \in E(S_i, l_j)$ and $(s_i, l_j, s_k) \in r_2$ (or $r_1$), then let $(S_i, L_j, S_k) \in R_2$ and go to step 4.
If $S_i \in E(S_k, l_j)$ and $(s_k, l_j, s_i) \in r_2$ (or $r_1$), then let $(S_k, L_j, S_i) \in R_2$ and go to step 4.
Otherwise, stop.

Step 4. Let $S=S \cup \{S_k\}$. If the cardinal number of S is equal to the number of regions on the line drawing D, then stop, otherwise go to step 2.

## V. General solution form and the use of gray level information

In Section IV, we have completed the quantitative shape recovery of a polyhedron corresponding to a given line drawing. It is described by a set of points S in the dual space D(f). However, the determination of S depends on four parameters. The reason is that in step 1, the initial point $S_1$ is selected arbitrarily within a region $C(s_1)$, and $S_2$ is selected arbitrarily on a line passing through the points $S_1$ and $[l_k]$ already known, so the former depends on three parameters and the latter depends one parameter. It is not surprised that Procedure 1 will get different solutions if the initial $S_1$ and $S_2$ are selected differently. This phenomenon reflects the fact that a line drawing may correspond to many objects in $R^3$. For example, two different objects ABCDEFG and A'B'C'DEFG shown in Fig. 6 will generate the same line drawing. In this section, we give a general solution form to represent the polyhedrons which are possible to generate a given line drawing.

Notice that the set of points S in space D(f) are determined by the operators $P_1$, $P_2$ and $P_3$ based on some colinear or coplanar relations. In order to obtain the general solution form, we consider a transformation denoted by $\tau$ in space D(f) such that the following conditions are satisfied:

1. $\tau$ preserves the colinear and coplanar relations between the points of L and S.
2. $\tau$ keeps the points of L unchanged.

Then after transformation, the output of S is still a solution of the quantitative shape recovery problem for the given line drawing.

From condition 1 above, it is easy to see that $\tau$ is a homogeneous linear transformation. And condition 2 shows that an invariant subspace exists. Now, we augment D(f) to a homogeneous space such that a point (p,q,d) in D(f) corresponds to a point (p,q,d+1,f) in it. Let $S=\{S_i | S_i=(p_i,q_i,d_i),\ i=1,2,\cdots,m\}$ be the initial solution obtained by Procedure 1, and the corresponding point set in the augmented space be $\{S_{ia} | S_{ia}=(p_i,q_i,d_i+1,f),\ i=1,2,\cdots,m\}$, then the transformation $\tau$ can be written as

$$\tau = \begin{pmatrix} 1 & 0 & \lambda_1 & 0 \\ 0 & 1 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & \lambda_4 & 1 \end{pmatrix}$$

It can be proved that $\tau S'_{ia}$ , $i=1,2,\cdots m$ in usual gives the general solutions to the quantitative shape recovery problem of a line drawing, where $\lambda_1$, $\lambda_2$, $\lambda_3$ and $\lambda_4$ are parameters to be determined.

In order to determine these parameters, it is needed to utilize some additional information. One of them is the use of gray level information whenever the line drawing is obtained from a gray level image. Please refer to [6] for the detailed formulas.

For example, Fig. 7 is a picture of a polyhedron taked under a natural illumination. Consider the line drawing extracted from Fig. 7. By executing Procedure 1, an initial solution can be obtained as $S_1,\cdots,S_8$, where the first five surfaces are visible, i.e. $S=\{S_1,\cdots,S_5\}$. A special solution obtained by above method is shown in Fig. 8. After that, we can get the gray level images of the object in different view directions artificially, and the results are shown in Fig. 9 . Obviously, these are in accord with the perception of human vision.

Finally, one can refer to [6] for an extensive discussion about the quantitative shape recovery of a complex polyhedral scene, and the approach for handing errors existed in a real image. It is also proved in [6] that the four basic constraints given in Section IV are the necessary and sufficient conditions for the existance of a polyhedron corresponding to a given line drawing.

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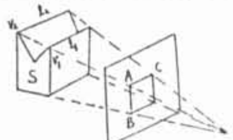
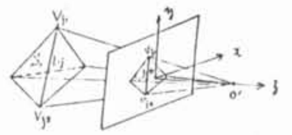


Fig.1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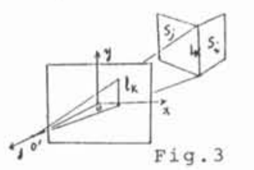
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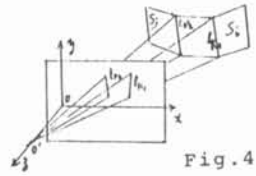
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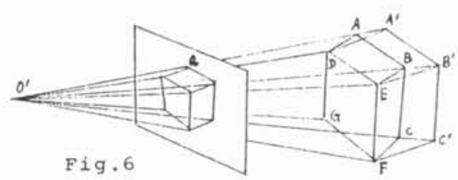
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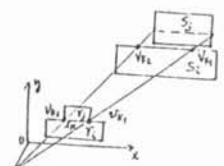
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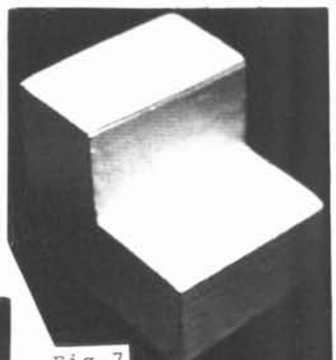
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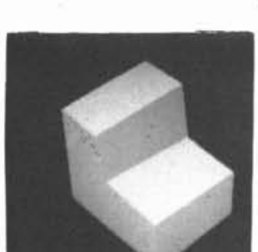
Fig.8

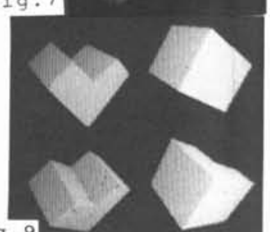
Fig.9