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Bi-Direction ICP: Fast Registration Method of Point Clouds

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Abstract

Iterative Closest Point is a vital method in registration of point clouds. It runs slowly in case of large point clouds. In this article, we propose the Bi-Direction ICP in a 2D plane. The rotation model is used to complete Bi-Direction ICP in 3D space. Bi-Direction ICP decreases the iteration times by expanding each step angle width. This method concentrates on shortening the running time of ICP without losing the accuracy. Applied Bi-Direction ICP into Stanford 3D Scanning Repository and Mian's datasets, the registration running speed is 2.7 times at most of that in ICP. A subtle improvement occurs in the aspect of accuracy, compared to ICP.

1. Introduction

In registration of point clouds, it merges the model point set and moving point set into a point set. Iterative Closest Point is proposed by Besl and McKay [1]. In their method, a point in model point set and the closest point in the moving point set form a point-pair. The rotation matrix and the translation vector between the point clouds are get by minimizing the energy function. The energy function is the sum of the L2 norm of the point-pairs.

Despite of ICP's simplicity and good performance in practice, ICP becomes slow while the number of points goes up in the point clouds. To reduce the running time of ICP, papers are put up with many methods to search the point-pair fast and robust. Jon Louis Bentley [2] develops the k-d tree method to search the nearest point. Michael Greenspan and Mike Yurick [3] present a depth-first non-tentative search to the k-d tree structure. Timothee Jost and Heinz Hugli [4] propose heuristic closest point search method to decrease the complexity of searching the closest point. They extend the number of points to 30000 [4]. Instead of finding the search fast and robust, these methods still spend much time while the iteration times don't change. In this paper, we proposed the first method to reduce the iteration times to get a faster speed than ICP.

There are other ways to optimize the running speed. S. M. Yamany, M. N. Ahmed, E. E. Hemayed and A. A. Farag [5] proposed the grid closest point transform to get a fast registration of point clouds. In their method, it needs to calculate the grid to search the closest point.

C.A. Kapoutsis, C.P. Vavoulidis and I. Pitas [6] put up with Voronoi tessellation method under Euclidean distance metric. Meanwhile, Voronoi map is a dual of Delaunay triangulation. Meanwhile, k-d tree search is replaced by Delaunay triangulations in the paper [7]. D. Eggert and S. Dalyot [7] use the Delaunay triangulation to increase computation efficiency in handling of data with ICP. With the use of triangulation in the Stanford bunny point cloud, they get an accurate result of registration.

In this paper, we wanted to reduce the iteration times we spent on registration. Since the formers contribute to getting the closest points fast and precisely in each step, they neglect that too many iteration times run in ICP before it stops. We focused our method on reducing the iteration times to get fewer iteration times. In 2D plane registration, we registered the point clouds in bi-direction and moved the moving point set with a merged rotation matrix of the bi-direction rotation matrixes. However, when it comes to 3D space, it may be trapped into a local dilemma. To solve this, we set up a rotation model to analyze 3D point cloud registration and use Principle Component Analysis (PCA) to do coarse registration. Then, we completed our method.

In our experiment, ICP was realized with Delaunay Triangulation and SVD. We applied ICP into Stanford 3D Scanning Repository [8] and Mian's datasets [9] to get a comparison to our methods. Based on this, we get the Bi-Direction ICP and apply it into the former datasets. Compared the results of our methods with the results of ICP, we get a fast method on running speed and a subtle improvement on accuracy.

We introduced the optimization of iteration times first and then, with the use of Delaunay triangulation and SVD, it turned out to be an accurate result.

In [12], Shaoyi et. al. improved the energy function with a bi-directional method to integrate the differences of ICP in two directions, which has no contributions to speed-up. Our bi-direction method is applied in the ICP process to expand the angle width of each ICP step, which shortens the time of registration.

This paper is organized as follows. Section 2 begins with an overview of the ICP work. Section 2.1 and 2.2 explain the ICP and the rotation angle in each iteration. Section 3 illustrates our method. Section3.1 and 3.2 explain our method in 2D plane and 3D space. Section 3.3 supplements the iteratively reweighted least square (IRLS) method [10] to gain an accurate result. Section 4 illustrates the experiment results of our method and ICP. In Section 5, conclusions are drawn and future work is discussed.

2. Iterative Closest Point (ICP)

Besl and McKay propose the Iterative Closest Point (ICP) to merge the model point cloud and the moving point cloud [1]. Suppose the model point cloud $M=\{p_i, i=1,2,\dots|M|\}$ and the moving point cloud $N=\{q_j, j=1,2,\dots|N|\}$, and what we need is to find the pair point of p_i in N.

2.1. ICP in Cartesian Coordinate System

For each point p_i, we need to find the closest point q_i in N and p_i-q_i forms a point-pair. Then, we get a set of point-pairs. The correspondence point-pairs contribute to the energy function with the rotation matrix R and translation vector T: $E=sum((p_i-q_i')^2)/|M|$, where the transform equation $q_i'=Rq_i+T$. E is also named as a distance function.

When the E is the minimum, the R and T meet the demand. The optimal equation is:

$$(R,T)=arg_{(R,T)}\ min\ E \qquad (1)$$

Once the point-pairs are found, we can get the R with singular value decomposition (SVD) and T with the mean translation of the closest point set. The rotation matrix is essential in Bi-Direction ICP.

2.2. ICP with Quaternions

Berthold K. P. Horn [11] presents a closed-form solution of absolute orientation using quaternions. Let Q take the place of the related quaternions, which shows:

$Q= q_r+q_i i+q_j j+q_k k$ and,

$$\boldsymbol{\theta}=2\arccos q_r=2\arcsin \mathrm{sqrt}(q_i^2+ q_j^2+ q_k^2) \qquad (2)$$

shows in Figure 1.

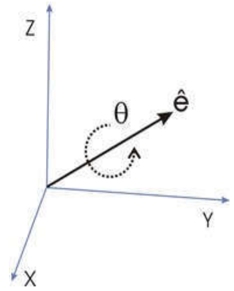

Figure 1: $\hat{e}$ spins by the angle $\boldsymbol{\theta}$.

The related rotation matrix is:

$$R=\begin{bmatrix} 1\text{-}2q_j^2\text{-}2q_k^2 & 2(q_iq_j\text{-}q_kq_r) & 2(q_iq_j+q_kq_r) \\ 2(q_iq_j+q_kq_r) & 1\text{-}2q_i^2\text{-}2q_k^2 & 2(q_jq_k\text{-}q_iq_r) \\ 2(q_iq_k\text{-}q_jq_r) & 2(q_jq_k\text{-}q_iq_r) & 1\text{-}2q_i^2\text{-}2q_j^2 \end{bmatrix}$$

By this way, the rotation matrix in ICP process can be shown with the rotation angle $\boldsymbol{\theta}$. Angle error also judges the result of registration.

3. Bi-Direction ICP (bi-ICP)

With the rotation angle in each ICP iteration, we can get a set of angles. Suppose the angle set $S=\{\boldsymbol{\theta}_i\ ;i\in\mathbb{N}^*\}$,

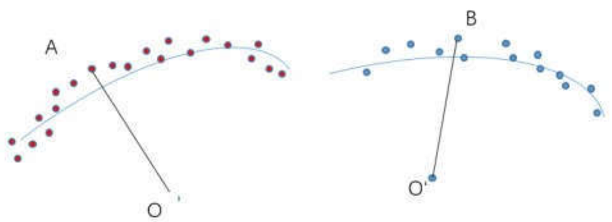

Figure 2: the red points and blue points belong to two point clouds. O and O' are the centers of the two sets. A and B are a true point-pair.

the i means the i-th iteration in the ICP. We discuss the variation of S in this part. Moreover, we apply the Bi-Direction in the plane and 3D space with a rotation model.

3.1. Bi-Direction ICP in 2D Plane

When the model point set and the moving point set are in the 2D plane, the rotation matrix is in a simple form. The form is $R=[\cos\boldsymbol{\theta}\ \ \text{-}\sin\boldsymbol{\theta};\ \sin\boldsymbol{\theta}\ \ \cos\boldsymbol{\theta}]$. The registration process and effects are shown in Figure 2 and 3.

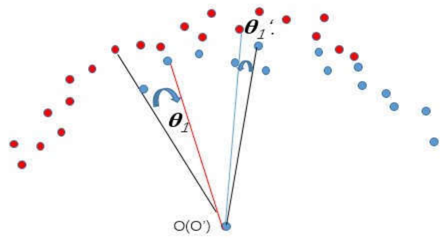

Figure 3: In the first iteration of ICP, A-pair and B-pair are shown by the closest line.

From the first iteration in ICP, we can get a minimum of energy function with rotation matrix $R=[\cos\boldsymbol{\theta}_1\ \ \text{-}\sin\boldsymbol{\theta}_1;\ \sin\boldsymbol{\theta}_1\ \ \cos\boldsymbol{\theta}_1]$.

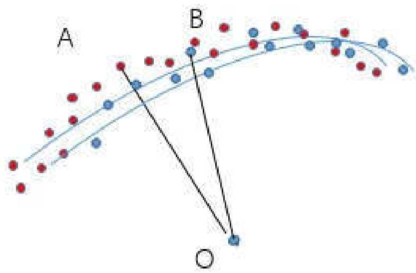

Figure 4: Moving set moves by the angle of $\boldsymbol{\theta}_1+\boldsymbol{\theta}_1'$.

Compared to the direction from model set to moving set, we invert the registration direction and get $\boldsymbol{\theta}_1'$. In the angle set S, with the ICP method, we can get: $\boldsymbol{\theta}_1\geq\boldsymbol{\theta}_2\geq...\geq\boldsymbol{\theta}_i\geq...$ and $lim_{n\to\infty}\ \boldsymbol{\theta}_n=0$. To increase the step angle, we add the angle $\boldsymbol{\theta}_1$ to $\boldsymbol{\theta}_1'$ and the step width spread to $\boldsymbol{\theta}_1+\boldsymbol{\theta}_1'$. After that, the OA and the OB are registered in first iteration and the rotation matrix is $R=[\cos(\boldsymbol{\theta}_1+\boldsymbol{\theta}_1')\ -\sin(\boldsymbol{\theta}_1+\boldsymbol{\theta}_1');\sin(\boldsymbol{\theta}_1+\boldsymbol{\theta}_1')\ \cos(\boldsymbol{\theta}_1+\boldsymbol{\theta}_1')]$. By this way, we get the step angle as $\boldsymbol{\theta}_1+\boldsymbol{\theta}_1'$.

By this way, we spread the step width of the iteration of ICP. Applied it into the other iterations, we finished the Bi-Direction ICP in 2D plane.

3.2. Bi-Direction ICP in 3D Space

Bi-Direction ICP is shown to act on 2D registration in section 3.1. However, it can't be extended into 3D space simply, since the angle direction becomes more complex and the rule of angle addition in 3D space has changed compared to 2D plane. To solve this, we modify the bi-Direction ICP to adapt to the change of the rule and make Bi-Direction ICP suitable to 3D registration as well.

As it shows in Figure 1 and Section 2.2, the model point cloud and the moving point cloud can be aligned by rotating around a spinning axis. So, based on the spinning axis, we can get a rotation model illustrated in Figure 5.

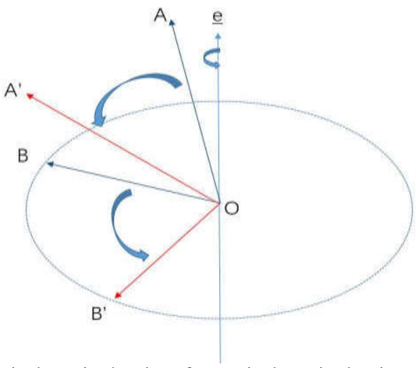

Figure 5: A' is the paired-point of A; B' is the paired point of B. e is the spin axis.

In the Figure 5, we divide the rotation model into 3 parts: point-pairs on the spinning axis, point-pairs on the plane vertical to the spinning axis and the other point-pairs. The ones on the spinning axis are no longer discussed since they

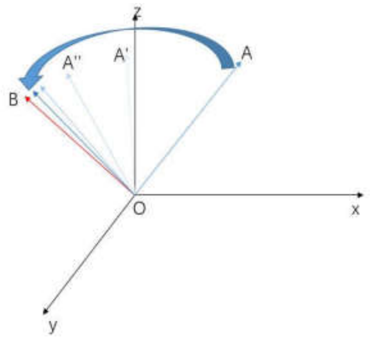

Figure 6: OB is the target. OA moves by rotating around different spinning axis and finally is paired with OB.

only matter the translation vector. The ones on the vertical plane are the same as the process in 2D plane except one dimension has been added to the expression of the matrix R.

So the third part in our essential part needs to be analysis. The paired points in this part are not in a same plane vertical to e in each iteration of ICP. In the 3D space, the path of ICP registration is shown in Figure 6.

The biggest challenge for Bi-Direction ICP is hinted in Figure 6. The angle of bi-direction rotation matrix is no longer an addition of two direction angles. The integration of the two direction rotation matrix is more complex and uncertain.

To solve the addition problem, we brought the rotation matrix set $M=\{R_i; i\in\mathbb{N}^*\}$ and $M'=\{R_j; j\in\mathbb{N}^*\}$ on the opposite registration direction. Suppose the number of elements in M and M' are m and n.

Definition: if the start point is B, we get S'. We inverse the process by $R_n, R_{n-1}, \ldots, R_1$ in S. Then in each step, B^i links B with a distance. Based on the center B and BB', we can get a dome in the sphere O. Consider the rotation matrix in S', we can get the B' after R_1' in S' and BB'. If $|BB'|>|BA^{i+1}|$ and $|BB'|<|BA^i|$, we call that R_1' **covers** $R_n, R_{n-1}, \ldots, R_{i+1}$ in S.

With the definition and Figure 7, an iteration in Bi-Direction ICP **covers** many iterations in ICP. The more elements it **covers**, the less iteration times it runs with Bi-Direction ICP. As shown in the Figure 1, the second iteration in Bi-Direction ICP **covers** the arc A'C' rotating around the line OB on the sphere surface (between the two gold disks). Now, we successfully

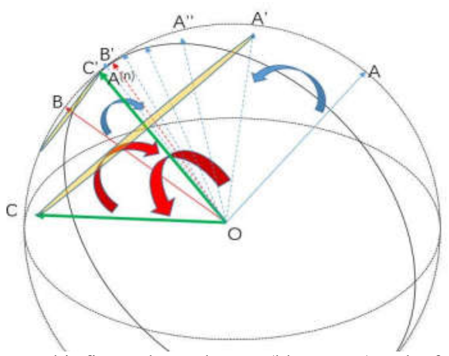

Figure 7: This figure shows the ICP (blue arrow) path of OA→OB and the Bi-Direction ICP (red arrow) path of OA→OC→OC'…→OC^i→..→OB.

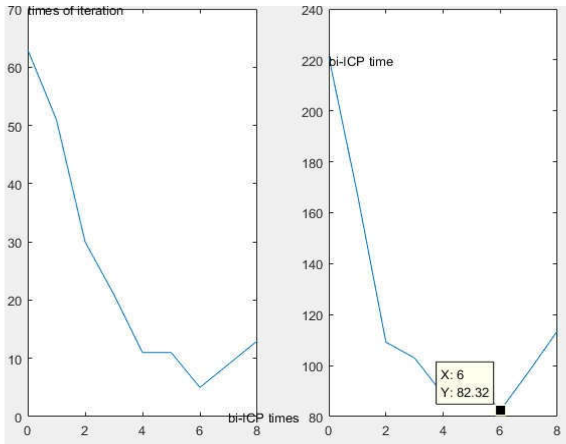

Figure 8: The running time and the iteration times of Bi-Direction ICP go with the bi-direction iteration times going up.

reduce the times of iteration in ICP with bi-direction method.

However, the bi-direction method doesn't guarantee a good convergence, which the ICP method could make. By this way, we combine the bi-direction method and ICP method together to acquire fast registration and good convergence.

We applied Bi-Direction ICP into the registration of Buddha, which comes from the Stanford 3D Scanning Repository [8]. In the Figure 8, the iteration times means how many times there are in all in the registration and Bi-Direction ICP time is the time that the whole process costs. The Bi-Direction ICP times means how many Bi-Direction iteration times only there is in the registration. In this experiment, we get the minimum of running time: Bi-Direction ICP times is 6 and the minimum time is 82.32s, while the running time of ICP is 221.28s. In this case, we manage to decrease the iteration times from 63 of ICP to 13 with the bi-direction method. Using bi-direction method with ICP, we reduce the running time of registration by skipping iterations successfully. Our experiment is followed in Section 4.

4. Experiment

In this section, we discuss a vital problem we met to solve the local minima of ICP. Based on the data set, we discover the ICP converges easily to local minima as the rotation angle between the two point clouds goes up. In our experiment, the local minima occurs when the rotation angle of the moving point cloud and the model point set is

120°~240°. So in order to solve this, we use principle components analysis (PCA) to match the eigenvalues of covariance matrix of two point clouds. Suppose the covariance matrixes of the model point cloud and moving point cloud are Cov_1 and Cov_2, and $Cov_1 = V_1\Lambda_1 V_1^T$, $Cov_2 = V_2\Lambda_2 V_2^T$ ($I=V_1V_1^T=V_2V_2^T$). The Λ_1 and Λ_2 are diagonal matrix with the form [λ_1 0 0;0 λ_2 0;0 0 λ_3] ($\lambda_1>\lambda_2>\lambda_3$). With this method, we let the initial rotation matrix $R=V_1V_2^T$. Through this method, we manage to eliminate the mismatch caused by the front and back inversion.

Dataset	Bunny	Dragon	Buddha
Origin	0.002	0.0018	0.0022
Huber	4.20e-4	3.34e-4	3.78e-4
Tukey	3.52e-4	3.46e-4	2.17e-4
Cauchy	3.92e-4	3.49e-4	2.76e-4
Welsch	3.60e-4	3.38e-4	2.20e-4

Table 1: The registration accuracy under IRLS with the datasets in Stanford 3D Scanning Repository. Origin means the root of the mean square. The unit of the RMS is meter.

Another problem is the lack of accuracy of the sum of least square. To solve this, we bring the IRLS [10] into our method. We apply all the IRLS weights into the point clouds of Stanford 3D Scanning Repository and get Table 1.

After solving the problems, we complete the process for registration. We apply the Bi-Direction ICP method into all the point clouds in Stanford 3D Scanning Repository and Mian's datasets. The comparing experiment is based on ICP with all the point clouds above. In the end, we get the merged point clouds and Table 2. In Table 2, Bi-Direction ICP shows better RMS and running time as it is expected.

5. Conclusions and Future Work

We propose the Bi-Direction ICP method to accelerate the registration speed and analyze the Bi-Direction ICP in 2D registration and expanded to 3D registration. In the experiment, we manage to reduce the running time of ICP by reducing the iteration times with bi-direction method. And as a complement, we solve the local minima of ICP with PCA matching, which makes the Bi-Direction ICP function a more robust method.

We have reduced the iteration times of ICP, but to guarantee the accuracy, we seek for the help of ICP. The later study is expected to concentrate on the convergent analysis of the algorithm.

6. Acknowledgement

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Dataset	Partial overlap (ICP and Bi-Direction ICP)						
	Size of point set		ICP (DT+SVD+IRLS)		Bi-Direction ICP		
	$\|M\|$	$\|N\|$	Time(s)	RMS(mm)	Time(s)	Ang.Err.	RMS(mm)
Bunny	40256	40097	71.90	0.356	68.37	0.7642°	0.352
Dragon	25914	25219	67.91	0.346	33.41	0.1402°	0.346
Buddha	78056	75582	221.28	0.218	82.32	0.0353°	0.217
Chicken	29518	30165	94.12	0.3045	90.38	---	0.3040
Parasauro	46006	42940	100.27	0.2966	80.86	---	0.2965
T-rex	38776	40214	123.04	0.2922	67.61	---	0.2919

Table 2: The size of point set means the number of points in a point cloud. Ang.Err. is short for angle error. '---' shows hard to follow due to the lack of the real angle. The RMS is gained with the Tuber IRLS method.