# Detecting Rotational Symmetry via Global/Local Image Analysis

Inagaki Yousuke Tanaka Kanji University of Fukui 3-9-1, Bunkyo, Fukui, 910-8507, Japan e-mail tnkknj@u-fukui.ac.jp

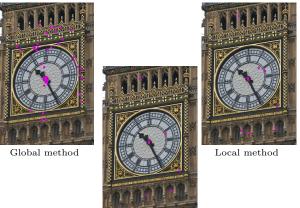
## Abstract

We propose a unified framework for rotational symmetry detection that combines the advantages of global and local image analysis. While existing methods based on global image analysis have proven to achieve stateof-the-art results, potential symmetries detected by the global methods still involve many false positives. Our idea is to employ efficient local image analysis as a cue, to intelligently choose most likely symmetry candidates, out of the potential candidates detected by the global method. We cast the sub-task of local image analysis as a correspondence growing task and introduce a robust solution employing an MCMC sampling algorithm. We demonstrate improvements over the best known method on challenging real images.

# 1 Introduction

One of major challenges in rotational symmetry detection comes from dealing with the appearance variations of the target symmetry. Since the variations are attributed to various factors (e.g., camera viewpoint change, partial occlusions, deformation of target objects) and cannot be seen beforehand, especially when dealing with natural cluttered scenes, detecting a skewed rotational symmetry inherent in real images is difficult.

Many efforts have focused on rotational symmetry detection using local or global image analysis methods [1]-[4]. Local feature-based methods detect symmetries from a set of selected key points, while global region-based methods detect symmetries using all pixels of an image. Local methods have the advantage of being more efficient for rotation symmetry detection. [1] proposed a pairwise local feature matching algorithm using SIFT key points at corresponding locations. The global methods, on the other hand, provide a more complete characterization of all potential symmetries in the image. [3] proposed a global regionbased method for Frieze-expansion (FE) and frequency analysis that calculates a Rotation Symmetry Strength (RSS) map for all pixels in an image. However, there is relatively little literature on dealing with skewed symmetries, which are inherent in real images. The most successful scheme reported to handle the skewed symmetry, thus far, is based on a global image analysis, and transforms an input image via frieze-expansion where the rotation center on the circular band becomes translation symmetries on a frieze-expansion pattern [4]. However, even the best methods do not yet distinguish many of true symmetries from background objects, and yield a large amount of false positives.



Hybrid method (proposed)

Figure 1. Results of COR (center of rotation) detection using the global, the local and the proposed hybrid methods. Only the proposed method achieves high detection ratio simultaneously with good detection accuracy.

In this paper, we revisit the local method and make contributions that improve over the best known method, the global method in [4] (Fig.1): (1) We develop a novel local method by employing an advanced local image analysis; (2) We use the developed local method as a cue to intelligently choose most likely symmetry candidates out of the potential candidates output by the global method; (3) We show via experimental comparisons the proposed method achieves high detection ratio while simultaneously with good detection accuracy.

The motivation of this study lies in the realization of a limitation of existing local methods: they are influenced by feature detection accuracy, and cannot exploit further information of the detected features. To overcome the limitation of the previous local methods, we adopt the image analysis based on correspondencegrowing which was originally studied within an alternative application of skewed reflection symmetry detection in [5]. In our rotational symmetry detection task, we efficiently explore the image space to exploit their information beyond the detected symmetric features, where consistent symmetric feature pairs are directly detected in the growing process without conventional voting procedure, such as Hough transform or RANSAC. Furthermore, we cast the correspondence growing task as an MCMC-based framework and use it as a cue to evaluate the likelihood of each symmetry candidate, which has been detected by the global methods.

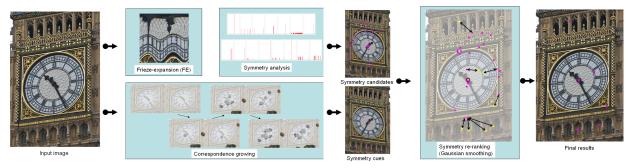


Figure 2. Detecting skewed rotational symmetries via global/local image analysis. The global analysis (the top pipeline in the figure) transforms an input image via frieze-expansion (FE), and then analyzes the center of rotation (COR) on the circular band which should be translation symmetries on a frieze-expansion pattern. The local analysis (the bottom pipeline) hypothesizes the center of rotation location, and then searches for further local features that support the COR hypothesis. The proposed method re-ranks the CORs output by the global method by using the CORs from the local method as a cue to intelligently choose most likely rotation centers.

# 2 Approach

## 2.1 System Overview

In general, there are five independent properties of a skewed rotation symmetry group, (1) the center of rotation, (2) the affine deformation, (3) the symmetry type, (4) the cardinality, and (5) the supporting group. In particular, the center of rotation (COR) property is an essential property, on which the other four properties heavily rely on, and we focus on detection of this COR property reliably and accurately.

Our COR detection approach consists of the global image analysis and the local image analysis subtasks (Fig.2). The former aims to provide a complete characterization of all potential CORs in the image by typically analyzing the global image structure and spatial frequency analysis. The latter local method aims at intelligently choosing true CORs out of the potential candidates output by the global method. The local method is designed to employ a correspondence growing algorithm that has been widely used in various types of image matching applications (e.g. object recognition, wide baseline stereo, image retrieval) and proven to be robust against changes in scales, viewpoints, partial occlusions and illumination.

Our local method consists of several different steps. First, it begins by hypothesizing the COR as one random pixel on the input map. By analyzing the neighbor region close to the hypothesized COR location, it searches for further local features that support the COR hypothesis. Second, we use a COR hypothesis verified by the local method to estimate the likelihood of neighbor COR candidates output by the global method. More detail, we employ a Gaussian smoothing operator, i.e. a 2D convolution operator, to compute the likelihood that a true COR exists at each location, and re-rank all COR candidates according to the likelihood computed. Third, we cast the local image analysis task as a probabilistic framework using an MCMC correspondence growing algorithm and formulate it in a comprehensive manner.

In the rest of this section, we explain the global method and the local method employed in this work in detail.

# 2.2 Global Method (The Baseline Method)

Our implementation of the global symmetry follows the known best method in [4]. The global method interprets the complex rotational symmetry detection problem as a simple translation symmetry problem by expanding a frieze-expansion (FE) method that transforms rotational symmetry group detection into a single, one dimensional translation symmetry detection problem. The existence of a rotational symmetry is usually supported by an annulus with or without a connected center point. If there is rotational symmetry about the center, this annulus contains repeating patterns along its circular direction (Fig.2). Given the location of a candidate rotation center, a diameter and a polar angle-step size, we re-align each diameter in parallel, from left to right, to form a horizontal pattern while advancing about the center angularly in the original image. If the candidate center location in the original image is indeed the center of a rotational symmetry, the converted frieze-expansion image becomes a true frieze pattern with non-trivial horizontal translation symmetry, and vice versa. The frieze expansion is performed for different possible settings of the aspect ratio of an image and all the potential COR candidates for all possible aspect ratio settings are output as COR candidates. In implementation, we consider aspect ratio settings that are represented by  $(1 + 0.2i)^j$  where i = 1, 2, 3, 4 and j = -1, 0, 1.

#### 2.3 Local Method

We adopt the local method based on correspondence growing, which was originally proposed for an alternative application of reflection symmetry detection in [5]. The correspondence growing process begins by the finding of tentative correspondences between pairs of the original input image and its mirrored image. A pair of SIFT descriptors is regarded as a tentative correspondence if the L1 distance between the normalized SIFT descriptors is smaller than 0.4. One known limitation of such tentative correspondences is that the size and the shape of the correspondence region is small and fixed. This involves a compromise. That is, too small correspondence region cannot capture global spatial property of an object while too large correspondence regions will include signal from background. A better estimate of correspondence quality can be obtained by concurrently looking at both the pair of regions in the input and the mirrored image by attempting to expand the correspondence domains.

In our implementation, the correspondence growing strategy for each step of iteration, either of two moves are possible: (1) correspondence initialization and (2) region expansion. In the first case, a new correspondence of SIFT interest points is established and a correspondence region pair is initialized. In the second case, a *frontier node*, a node at the boundary inside the current region within the query image is randomly sampled, then one of its neighbor nodes that lie outside the region, and as a next candidate of the expansion attempt (Fig.2). A pre-defined parameter  $\omega = 0.5$  controls the ratio between the two moves: the first move is chosen with probability  $1 - \omega$ .

Consider that we have an original input image I and its mirrored image I', and model an image as a lattice with a set of  $N^O$  nodes and edges. A rotational symmetry hypothesis h is represented by a pairing  $(X^h, Y^h)$ , of the corresponding node labels  $X^h = \{x_j\}_{j=1,\dots,N^O}$ and a set of affine transformations  $Y^h = \{y_k\}_{k=1,\dots,N^O}$ between a corresponding node pair. A node label  $x_j$  is a binary indicator which labels the corresponding node as belonging to the symmetry or not. An affine transformation (i.e. rotation, translation, scaling, etc.)  $y_k$ maps the location of the k-th node on the input image I to the corresponding location on the mirror image I'. Using the above terminology, the problem of rotational symmetry detection is formulated as finding an optimal set  $Z = \{(X^h, Y^h)\}$  of symmetry hypotheses that maximizes an objective function in the form:

$$p(Z|I) = p(I|Z)P(Z).$$
(1)

We explain each term in (1) and the inference algorithm to compute the objective function in 2.4.

#### 2.4 Inference Algorithm

We consider P(Z) as a prior model for the symmetry, and predict it from the size and the shape of each symmetry

$$P(Z) = \prod_{i} \exp\{-R_i/R^O\} \exp\{-S_i/S^O\}.$$
 (2)

 $R_i$  is the number of nodes in the current symmetry region.  $S_i$  is to evaluate the similarity of the configuration of nodes in the regions between the input and the mirrored images, as an averaged L2 distance between each node in the input image and its correspondence node in the mirrored image.

We model p(I|Z) as a likelihood function based on similarity of appearance between the correspondence region of interest on the input image and its corresponding region (i.e. obtained via affine transformations) on the mirrored image. An averaged color distance  $D_i$  in the normalized RGB color-space is employed as the similarity measure:

$$p(I|Z) = \exp\{-D_i/D^O\}.$$
 (3)

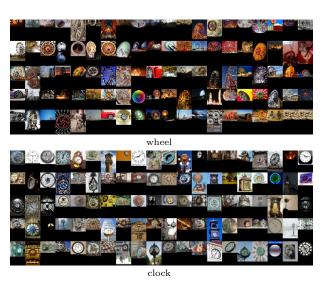


Figure 3. Dataset.

In the above expressions (2)-(3), symbols  $R^O$ ,  $S^O$  and  $D^O$  are pre-defined normalizer constants.

In order to sample in the high dimensional distribution represented by (1), some form of MCMC such as Gibbs are required. Our approach employs a simple MCMC scheme that will sample from the distributions with enough samples. A basic reversible ergodic MCMC that converges to a desired distribution can be constructed through the Metropolis-Hasting acceptance rule [6]. In addition to satisfying the irreducibility and aperiodicity properties, if samples are filtered by the following acceptance probability, the Markov chain satisfies the detailed balance condition, enforcing the sample distribution to converge toward the target distribution  $\pi(Z)$ .

$$A(Z',Z) = \min\left(1,\frac{\pi(Z')q(Z|Z')}{\pi(Z)q(Z'|Z)}\right)$$
(4)

This is the acceptance probability of going from state Z to Z'. q(Z|Z') is the proposal probability of going to state Z from state Z' before the acceptance probability is applied.

One of the drawbacks of the MCMC sampling scheme is the inefficiency due to its high rejection sampling rate. In order to overcome this problem, we employ a deterministic sampling strategy employed such as in [7] and other recent studies. In particular, we employ a strategy that accepts only those moves that expand or increase the size of correspondence region, and automatically reject those ones that decrease the sizes. Although more complex MCMC variants such as DDMCMC could be employed, the above simple modifications suffice in our framework to provide sufficient reliable detection of skewed rotational symmetries.

## **3** Experiments

The aim of this section is to investigate if the proposed method can outperform the baseline algorithm in rotational symmetry detection. To have an applicability of the proposed method, we have generated a generic query on Flickr, and searched two sets of size 100 image collections with the tag "wheel" and "clock" that belong to rotational symmetric objects. Then, we

		Data	
Algorithm	Wheel	Clock	
Global method	0.42	0.79	
Local method	0.20	0.42	
Proposal method	0.45	0.81	
Table 2. Averaged rank.			
	Data		

Table 1. Detection ratio.

	Data	
Algorithm	Wheel	Clock
Global method	181	101
Local method	11	15
Proposal method	154	87

manually prepared the ground truth of COR for each image and set the accept distance as 5 pixel. We have implemented three different methods for comparison. The first one is the baseline global method in [4] and in 2.2, called "global method". The second one is the local method described in 2.3, called "local method". The third one is the proposed hybrid method described in 2 that combines the advantages of the local and the global methods. To evaluate each method, we use detection ratio as the primary performance index. Detection ratio is the ratio of images where symmetry detection is successful to the total number of images. To further evaluate the performance on those images where the detection is successful, we use detection accuracy as the second performance index. Detection accuracy is the averaged rank of the true symmetry center detected out of the number of total symmetry centers detected. Tab.1 reports comparison of the detection ratio between the three methods. One can see that the proposed hybrid method clearly outperforms the other two. The global method tends to successfully detect potential symmetries and achieved high detection ratio, while the local method outputs much less number of symmetries that are supported by many local features and the detection ratio tends to be lower. The proposed hybrid method can combine both types of symmetries detected by those methods. Tab.2 reports the detection accuracy in terms of averaged rank, and Fig.4 shows random instances of the symmetry detection results for each of the three methods described above. While the local method achieved highest accuracy, as aforementioned the method outputs only small number of verified symmetries and yield low detection ratio. One can see that the proposed method can achieve high accuracy that outperform the global method, despite the fact that it can achieve highest detection ratio.

# 4 Conclusions

In this paper, we have proposed a robust and accurate approach for rotational symmetry detection, where symmetries are automatically detected via global / local image analysis. While the global method achieve state-of-the-art performance on detecting potential symmetry candidates, it also yields many false positives. To suppress such inaccuracy, we proposed to employ the efficient local method as a cue, to intelligently choose most likely symmetry candidates, out of the large number of potential candidates detected by

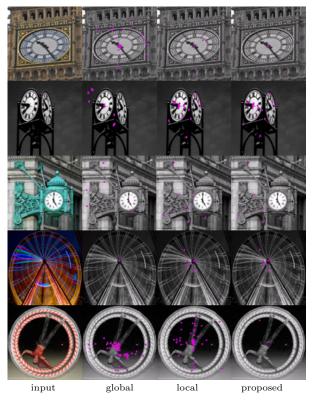


Figure 4. Symmetry detection results by three different methods.

the global method. In experiments, we have demonstrated the proposed hybrid approach outperforms the global and the local methods in terms of detection ratio and accuracy.

# Acknowledgement

This work was partially supported by MECSST Grant (23700229, 30325899), by KURATA grants and by TATEISI Science And Technology Foundation.

# References

- Gareth Loy and Jan-Olof Eklundh. Detecting symmetry and symmetric constellations of features. In ECCV (2), pages 508–521, 2006.
- [2] V. Shiv Naga Prasad and Larry S. Davis. Detecting rotational symmetries. In *Proceedings of the Tenth IEEE International Conference on Computer Vision - Volume* 2, ICCV '05, pages 954–961, Washington, DC, USA, 2005. IEEE Computer Society.
- [3] Seungkyu Lee, Robert T. Collins, and Yanxi Liu. Rotation symmetry group detection via frequency analysis of frieze-expansions. In CVPR, 2008.
- [4] Seungkyu Lee and Yanxi Liu. Skewed rotation symmetry group detection. *IEEE Trans. Pattern Anal. Mach. Intell.*, 32(9):1659–1672, 2010.
- [5] Minsu Cho and Kyoung Mu Lee. Bilateral symmetry detection via symmetry-growing. In *BMVC*, 2009.
- [6] Nicholas Metropolis, Arianna W. Rosenbluth, Marshall N. Rosenbluth, Augusta H. Teller, and Edward Teller. Equation of State Calculations by Fast Computing Machines. *The Journal of Chemical Physics*, 21(6):1087–1092, 1953.
- [7] Minsu Cho, Young Min Shin, and Kyoung Mu Lee. Unsupervised detection and segmentation of identical objects. In *CVPR*, pages 1617–1624, 2010.