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Fast ICP Algorithm Using a One-Dimensional Search

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Abstract

To automatically construct a three-dimensional (3-D) object model, it is necessary to merge a set of range images acquired from different views. The iterative closest point (ICP) algorithm has been widely used to find the transformation parameters between a pair of images.

In this paper, a fast ICP algorithm is proposed by restricting the search space to one-dimensional (1-D) orthogonal lines to find transformation parameters such as rotations and translations between the reference model and the object model.

The experimental result shows that the proposed algorithm is fast and accurate for estimation of the transformation parameters compared with the original ICP algorithm.

1 Introduction

Recent advance in 3-D scanner technology has made it possible to automatically construct 3-D object models. There are several methods to obtain range images: a laser triangulation method, time-of-flight or phase-shifting method, and a Moiré topography [1][2]. The application area of 3-D modeling has been enlarged due to the advance of 3-D scanner technology.

However, description of 3-D object surfaces with a single scanner image is incomplete because of the limit to the viewing angle in image acquisition. Therefore, multiple range images taken at different angles are to be merged, in which accurate estimation of transformation parameters between range images is needed.

Besl and McKay proposed the ICP algorithm [3] to find these parameters using an iterative least squares method, in which the point closest to each control point is selected as a matching point. The point-to-point matching method for finding the closest point by searching the whole space requires $O(N^2)$ operations, where N denotes the number of points in each range image.

The computational complexity of $O(N^2)$ is too high for real-time applications of 3-D modeling. In this paper, a fast algorithm to find a matching point is proposed by restricting the search space to 1-D orthogonal lines using a one at a time search (OTS) method [4]. The computational complexity is reduced to $O(N^{3/2})$, while the accuracy is guaranteed by eliminating outlier points due to the occlusion which occurs during the iterative procedure to find the transformation parameters.

2 Conventional Registration Algorithms

If the corresponding points between two range images are known, the transformation parameters can be found easily, but in most cases, it is hard to directly find the exact correspondences. In Besl and McKay's algorithm [3], the closest point is used as an approximate matching point, which may lead to an inaccurate transformation. However, as the method is an iterative one, it gives the correct solution if the distance from each control point to a matching point is decreased. The average squared distance error d_k^2 at iteration k is defined as

$$d_k{}^2 = \frac{1}{N_p}\sum_{i=1}^{N_p}\left\|q_{i,k} - p_{i,k}\right\|^2 \tag{1}$$

where p_i represents a control point in the object $\boldsymbol{P}$ to be registered and q_i denotes a matching point, in the model shape $\boldsymbol{Q}$, corresponding to p_i, with N_p signifying the number of data points in $\boldsymbol{P}$ and $\|\cdot\|$ representing a norm. The matching point q_i is selected as the point closest to the control point p_i among the points in the model $\boldsymbol{Q}$ to minimize the distance error.

The rotation angle can be calculated using the unit quaternion vector $\boldsymbol{q}_R$ [5] which is equivalent to the maximum eigenvector of the matrix $Q(\Sigma_{pq})$ defined as

$$Q(\Sigma_{pq}) = \begin{bmatrix} \mathrm{tr}(\Sigma_{pq}) & \Delta^T \\ \Delta & \Sigma_{pq} + \Sigma_{pq}^T - \mathrm{tr}(\Sigma_{pq})\boldsymbol{I}_3 \end{bmatrix} \tag{2}$$

where Σ_{pq} denotes the cross-covariance matrix, $\boldsymbol{I}_3$ is the 3×3 identity matrix, tr(·) represents the trace, the superscript T denotes transposition, and the cyclic components of the anti-symmetric matrix ($\Sigma_{pq} - \Sigma^T_{pq}$) are used to form the 3-D column vector Δ.

The translation vector is given by

$$\boldsymbol{q}_T = \mu_q - \boldsymbol{R}(\boldsymbol{q}_R)\mu_p \quad (3)$$

where μ_p and μ_q are the centers of mass of $\boldsymbol{P}$ and $\boldsymbol{Q}$, respectively, and $\boldsymbol{R}(\boldsymbol{q}_R)$ is the rotation matrix generated by a unit rotation quaternion vector $\boldsymbol{q}_R$.

After the transformation parameters are found, all points in $\boldsymbol{P}$ are transformed using those parameters and the closest points and transformation parameters are calculated again. The algorithm proceeds iteratively until the distance error converges and the convergence theorem of this iterative method is stated and proved by Besl and McKay [3].

The problem of the ICP algorithm is that it is too much time consuming. The computational complexity is the multiple of N_p, N_q, and k, where N_p and N_q are the numbers of points in the model $\boldsymbol{P}$ and $\boldsymbol{Q}$, respectively, and k is the number of iterations. If $N_p=N_q=N$, the order of computation can be represented as $O(N^2)$ because k is much smaller than N.

Besl and McKay proposed an accelerated ICP algorithm [3], in which the number of iterations is reduced. This algorithm improves convergence speed, but cannot be the fundamental solution to the high computational load, because the number of iterations is too small compared with the number of point-to-point matchings.

Chen and Medioni proposed a point-to-surface matching algorithm [6] which is very fast to find a matching point, but it needs approximate initial transformation values, without which the convergence cannot be guaranteed. Also a number of differential calculations are needed to find a line-surface intersection, which cause additional computational complexity and numerical instability.

An algorithm with logarithmic expected time [7] was applied to find the closest point. Zhang used a cyclic 3-D tree search algorithm [8] and Hügli *et al.* used a balanced 3-D tree search algorithm [9]. The computational complexity is reduced to $O(N^{5/3})$ and $O(N\log N)$, respectively, but the performance is highly dependent on the distribution of data set.

Another registration algorithm using a reverse calibration technique was proposed by Chung *et al.* [10]. Though it is fast, the calibration parameters of the 3-D scanner must be known beforehand, and it is applicable to some specific scanner devices.

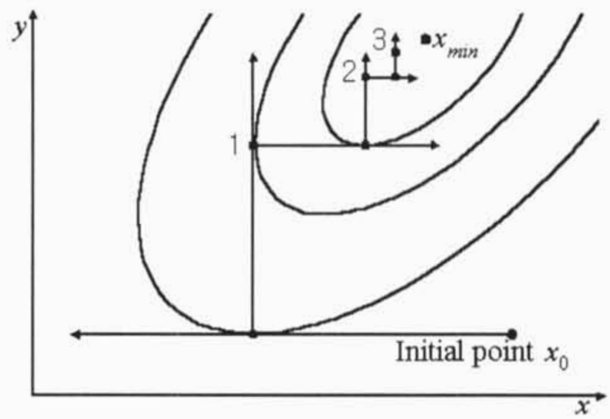


Figure 1: Example of OTS procedure for 2-D optimization function with 3-step adjustment.

Shum *et al.* proposed a 3-D modeling algorithm using a principal component analysis with missing data [11]. It is robust to noise and converges fast, but the application is restricted to an object with planar surfaces such as a polyhedral object.

3 Proposed Matching Algorithm

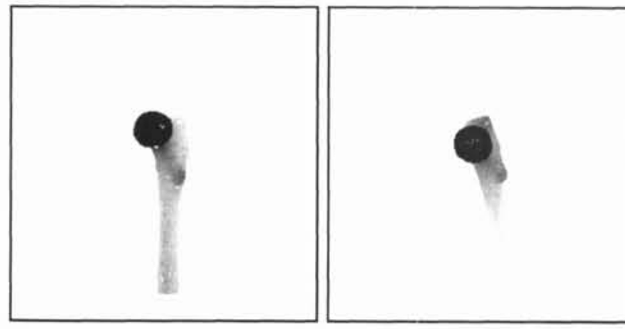
To find a point closest to $\boldsymbol{p}_i$, every point in $\boldsymbol{Q}$ should be considered. However, instead of the exact closest point, an approximated one can be used only if the convergence is guaranteed. The convergence is preserved whenever the distance between the matching points decreases [3].

For fast detection of the closest points, OTS method [4] can be applied, in which all the parameters are adjusted for optimization of the performance measure in turn, one at a time. This adjustment is repeated until the correct value is detected. Fig. 1 illustrates the adjustment procedure for a two-dimensional (2-D) optimization function with 3-step OTS, where each step involves a search along a pair of orthogonal lines that are parallel to the x- and y-axes.

To apply the OTS method to the ICP algorithm, a point that is projected along the z-axis from a control point to the object model is considered as an initial point, and the squared distance error expressed in (1) is used as an objective function to be minimized. If the size of the model in the image plane is $m\times n$, the computational complexity for point matching is reduced from $O(m\times n)$ to $O(m+n)$. Although this method is an iterative one, the required number of steps is small compared with $m\times n$ and the experimental results show that only two- or three-step OTS can produce good results.

To obtain more accurate results, we eliminate invisible points generated by the transformation parameters estimated at the current step. Because the two images are acquired at different viewing angles, occlusion may occur. When the model is transformed, some points may be invisible because they disappear behind the occluding part. Since they do not have corresponding points, the points closest to them are regarded as their matching points, which results in large matching error.

If the z component of the surface normal vector [12] at the point is negative, the point is considered as a hiding point. By eliminating hiding points (outliers), the incorrect matching points are successfully removed, yielding more accurate transformation parameters.



(a) (b)

Figure 2: A set of range images of a real human bone. (a) original model, (b) rotated one by 15° each along the x-, y-, and z-axes.

4 Simulation Results

The proposed algorithm is tested with several synthetic and real range images, one of which is presented to show the effectiveness of the proposed algorithm. The test range image shows a part of a real human bone, which was acquired with a Minolta VI-700 3D digitizer (obtained from http://www.3dscanner.ch). Fig. 2(a) shows the original model and Fig. 2(b) illustrates the transformed one by rotating the model in Fig. 2(a) by 15° each along the x-, y-, and z-axes. In Fig. 2, the horizontal direction, the vertical direction, and the direction normal to the x-y plane correspond to the x-, y-, and z-axes, respectively. Each pixel in the image has a range value and the depth is represented in gray level: the lighter the pixel, the deeper the depth.

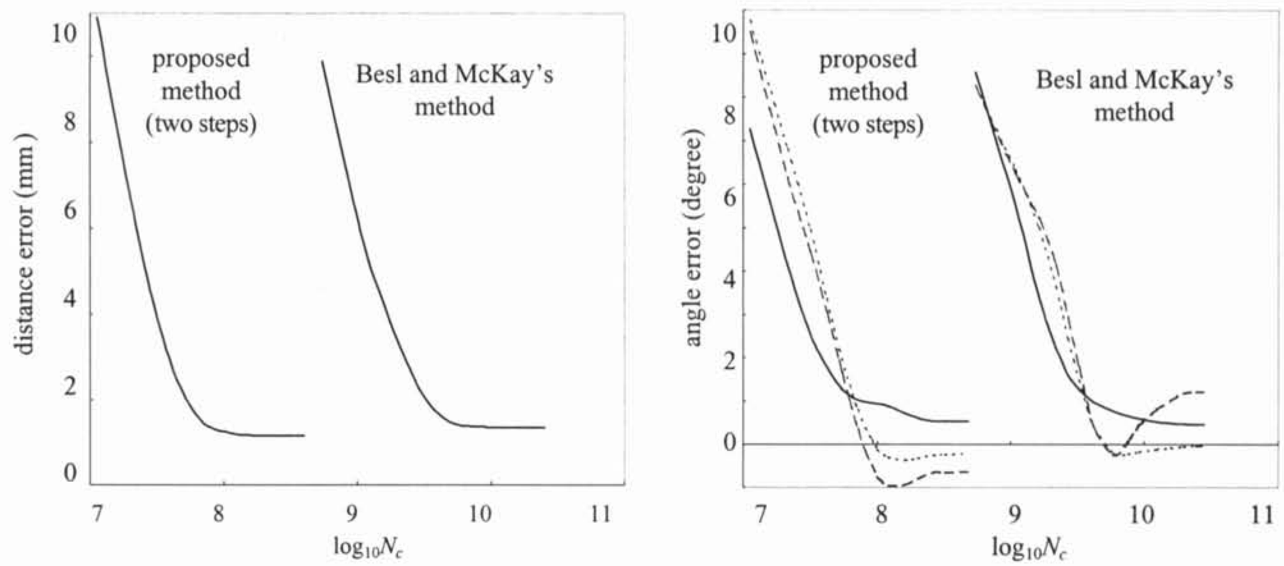
The original model shape looks like a short stick, so when the model is rotated along the x-axis, the number of hiding points increases, which yields inaccurate matching. When it is rotated along the y-axis, the shape is slightly changed, so the reliability of the ICP algorithm decreases [13]. In the case of the rotation along the z-axis, no such occlusion problem arises.

Experimental results are shown in Table 1, in which the performance of the proposed method with up to five OTS steps is compared with that of Besl and McKay's method [3]. Table 1 lists the distance and rotation angle errors. The sampling distance between points in the x-y plane is assumed to be 1 mm. In the proposed method, as the number of OTS steps increases, the error is reduced: even smaller than that of Besl and McKay's method, with greatly reduced computational complexity. This reduction in error is due to elimination of outlier points.

Figs. 3(a) and 3(b) show the distance and rotation angle errors as a function of $\log_{10}N_c$, respectively, in which the computational complexity for point matching is expressed in terms of the number of additions and multiplications N_c. Note that the 2-step OTS is adopted in the proposed algorithm. The convergence curves for the x-, y-, and z-axes in Fig. 3(b) have different characteristics, but they converge to the correct values. The solution of the proposed method is more accurate because the outliers are effectively eliminated. Moreover, the computational complexity is greatly reduced, which is shown in Table 1. The larger the number of data points in the model, the greater the computational reduction ratio.

Table 1: Distance and rotation error of the proposed method and Besl and McKay's method.

		Error				N_c ($\times 10^6$)
		Distance (mm)	Rotation angle (degree)			
			x	y	z	
Besl and McKay's method		1.365	0.477	1.227	-0.182	16382
Proposed method (number of steps)	1	1.213	0.708	-0.760	-0.425	151
	2	1.161	0.540	-0.618	-0.214	310
	3	1.160	0.545	-0.633	0.226	435
	4	1.160	0.547	-0.627	-0.220	579
	5	1.160	0.547	-0.627	-0.220	724



(a) distance error (b) angle error (—— x-axis, - - - y-axis, ···· z-axis)

Figure 3: Comparison of the computational complexity.

5 Conclusion

In this paper, a fast ICP algorithm is proposed, in which the search space for point matching is restricted to 1-D orthogonal lines by using the OTS method. The computational complexity is reduced from $O(N^2)$ to $O(N^{3/2})$, where N denotes the number of points in the range image. The matching performance is improved by eliminating the outlier points caused by occlusion.

Experimental results with several test images show the effectiveness of the proposed method in terms of the reduction of the computational complexity, while the accuracy is guaranteed.

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