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8—7 Structural Stability Analysis for Texture Recognition

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Abstract

The tree-structured wavelet transform has received a lot of attention and has found successful applications in signal denoising, image coding, image analysis, etc. In this paper, we present an analysis of the structural stability of the tree-structured wavelet transform, a topic which has not been addressed properly in previous research. We also present a texture classification algorithm based on a bottom-up tree-structured wavelet transform. The proposed approach does not require ad-hoc parameters and demonstrates superior performance, as compared to the top-down approach proposed previously.

1 Introduction

Texture recognition in digital images has received considerable attention over the past few decades and a large number of approaches have been suggested. Despite these efforts, texture analysis is still considered as an interesting but difficult issue in image processing and computer vision [1] [2].

One difficulty in texture recognition was the lack of adequate tools for characterizing textures. The wavelet transform attempt to overcome this difficulty, especially, the tree-structured wavelet transform (TSWT) has received a lot of attention. Recently, Chang et al. [3] proposed a texture classification algorithm based on a top-down TSWT. Although, high classification rates were reported, its performance is highly dependent on a set of parameters that have to be determined in an ad-hoc way.

In this paper, we provide an analysis of structural stability of TSWT, a topic that has not been addressed properly in previous researches. We also propose a texture classification algorithm based on a bottom-up TSWT, which is not dependent on ad-hoc parameters and shows superior performance.

2 Space-Frequency Decomposition Using Wavelet Transform

In the pyramid-structured wavelet transform (PSWT), the decomposition is performed recursively to the output of the lowpass filter. However, since the most important information of textures often appears in the middle frequency channels, further decomposition just in the lower frequency subbands may not help much for the purpose of texture recognition. A more effective way to perform the wavelets for textures is to find the significant frequency channels and then decompose them further.

The above idea leads naturally to TSWT [4]. The key difference between TSWT and PSWT is that in TSWT the recursive decomposition is no longer simply applied to the low frequency subbands. There are basically two approaches that can be used to extract the significant subbands of a texture image: the top-down ($\mathrm{TSWT_{TD}}$) and bottom-up ($\mathrm{TSWT_{BU}}$) approach. In $\mathrm{TSWT_{TD}}$, a texture is decomposed recursively from the top to destination level, while the decomposition is performed only on the subbands which are deemed to be important. On the other hand, in $\mathrm{TSWT_{BU}}$, a texture is first fully decomposed and then the subbands which are deemed to be unimportant are pruned, as the algorithm proceeds from the bottom to the top level.

Chang's algorithm (Figure 1) employs a criterion to decide whether a subband has to be decomposed or not. In the algorithm, only the nodes whose ancestors are significant will be considered as the candidates for further decomposition. This leaves the possibility of missing significant nodes that fail to have significant ancestors. Moreover, the algorithm classifies textures using only the features of the dominant subbands. In the following, we describe short-

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comings of the algorithm in more detail.

1. Decompose a given textured image by using TSWT into 4 subimages, which can be viewed as the parent and children nodes in a tree.
2. Calculate the energy of each decomposed image.
3. If the energy of a subimage is significantly smaller than others, we stop the decomposition in this region since it contains less information. That is, if $e < Ce_{max}$, stop decomposing this region where C is a constant less than 1.
4. If the energy of a subimage is significantly larger, we apply the above decomposition process to the subimage.

Figure 1: The TSWT_{TD} algorithm.

Firstly, since pruning is based on the local rather than global energy features TSWT_{TD} would not always result in a globally optimal space-frequency decomposition. To illustrate the point, suppose that among the four nodes in the level 1 (see Figure 2), the first node has the maximum energy and that the others have energies smaller than Ce_1. Here e_i denotes the energy of the i-th node. Then, if we decompose the nodes at the level 1 using TSWT_{TD}, only the first node will be decomposed and hence the children nodes under the other nodes will not be visited even if some of them might have large energies.

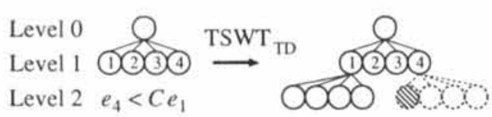

Figure 2: Example of a problem in TSWT_{TD}.

Secondly, they propose to characterize each texture by the energy map that is defined as the vector whose components are the energy of subbands appearing in the structure of a texture. However, since the structure is not unique for a texture, they modify all structures of a texture to the most diverse one and use the average of the corresponding energy maps as the texture's template.

Lastly, they employ energy maps defined over the first five dominant nodes rather than the whole nodes of a structure. Although, the adopted approach results in less structural variations of structures, it deprives the classification algorithm from the ability to discern texture samples which are identical with respect to their first five dominant nodes.

3 Texture Recognition by TSWT_{BU}

3.1 Decomposition and Structuring

In the proposed TSWT_{BU} based algorithm (Figure 3), since pruning proceeds from the bottom to the top level, all nodes will be visited and there is no danger of missing a significant node, which was not the case with Chang's algorithm. Furthermore, the algorithm is free from the ad-hoc and heavily data dependent parameter C, which is needed for making a trade-off between the structural stability and the classification accuracy in the algorithm of Chang.

1. Decompose a given textured image fully by using TSWT.
2. Calculate the energy of each decomposed node.
3. If the energy of a parent node is larger than the average of that in its children, we prune the children.
4. Repeat step 3 from the bottom level to the top.

Figure 3: The TSWT_{BU} algorithm.

The algorithm, fully decomposes a texture sample by TSWT and then calculates the energy of the nodes of the sample's fully-decomposed tree-structure. The energy of a node is defined as

$$e = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} |x(m,n)| \tag{1}$$

where M and N denote the number of the rows and columns of the subband corresponding to the node, and $x(m,n)$ denotes the wavelet coefficient at the (m,n)-th position. If the energy of a node is larger than the average of the energy of its children, the children are considered as being insignificant and are pruned. This pruning scheme proceeds from the bottom to the top and results in the sample's structure.

Ideally, it is desirable that each texture has a unique structure, and that these structures have large between-class scatter. However, most of textures shows structural diversity which makes it difficult to characterize them efficiently in terms of their structure or the corresponding energy templates. Energy template is defined as the average of the energy maps for all the samples of a texture which have identical structure. In the proposed algorithm, we first identify all of the distinct structures, found over the samples of a texture, and then calculate the corresponding energy templates (Figure 4).

1. Count the number of structures k_i in T_i.
2. Average the energy and generate a template for structure j.
3. Repeat step 2 for all structures in T_i.
4. Repeat step 1 - 3 for all textures.

Figure 4: Generating templates.

3.2 Structural Stability Analysis

One of the basic problems in characterizing textures using TSWT is the structural diversity. Though, Chang referred to this problem, they did not provide any measure for quantifying this diversity. We introduce an entropy based measure, called structural entropy.

First, We define the local structural entropy of the i-th texture as

$$H_{local}(T_i) = -\sum_{j=1}^{k_i} q_j \log(q_j) \tag{2}$$

where k_i denotes the number of structures of the i-th texture and q_j denotes the probability of occurrence of the j-th structure. It takes on value zero if all samples have identical structure. When multiple structures exist, local texture entropy would take on low values if a structure occurs very frequently and the other structures very rarely, and takes on high values if structures occur uniformly. We also define the global structural entropy as

$$H_{global} = -\sum_{l=1}^{n} p_l \log(p_l) \tag{3}$$

where n denotes the number of structures found over the samples of all textures and p_l denotes the probability of occurrence of the l-th structure.

Generally , it is desirable to have low local structural entropies and a high global structural entropy. Hence, it would be proper to define the structural entropy of a texture as the ratio of its local structural entropy to the global structural entropy as:

$$H(T_i) = \begin{cases} \frac{H_{local}(T_i)}{H_{global}} & (\ n \neq 1\ ) \\ 0 & (\ n = 1\ ). \end{cases} \tag{4}$$

The structural entropy of a texture will take on low values if the local structural diversity is low and the global structural diversity is high. $H(T_i)$ is defined to be zero when all samples have a unique structure, as in PSWT. Figure 5 shows the algorithm.

1. Count the number of structures (n) for all samples.
2. Calculate the probability p_l for all structures.
3. Calculate the structural entropy H_{global} for all samples.
4. Count the number of structures (k_i) for the texture T_i.
5. Calculate the probability q_j for all structures in T_i.
6. Calculate the structural entropy H_{local} in T_i.
7. Calculate the texture entropy $H(T_i)$.
8. Repeat steps 4 – 7 for all textures.

Figure 5: Calculating structural entropy.

3.3 Classification

Chang uses only five dominant nodes of a sample's structure for classification, so misclassification may occur when samples of different textures happens to have identical dominant nodes. To avoid this problem, we make use of the complete structure, rather than few dominant nodes (Figure 6).

- *Learning phase:*
  1. Decompose a training sample by using TSWT, and calculate the energy and structure vector.
  2. Repeat step 1 for all samples.
  3. Generate templates for all structures.
- *Classification phase:*
  1. Decompose a test sample by using TSWT, and calculate the energy and structure vector.
  2. Use the energy in the same nodes as the test sample from each template. If the same nodes do not exist, set the energy to zero.
  3. Calculate the distance by using the distance function.
  4. Assign the test sample to the texture that has the minimum distance.

Figure 6: Classification algorithm.

In the learning phase, we decompose the training samples by using TSWT_{BU} and construct the structure vectors and the corresponding energy templates for each texture. Structure vector is a binary vector that we use to encode a tree-structure. It is constructed by scanning the nodes in a tree-structure from the top to bottom level and assigning 1 or 0 to the vector coefficient according to whether the corresponding node is split or not to its children (see Figure 7). Since the lowest level has no children, the structure vector is uniquely determined by the first $L-1$ levels of the tree, where L denotes the wavelet decomposition level. Hence, the dimension of the structure vector is given by

$$d = \sum_{i=0}^{L-1} 4^i. \tag{5}$$

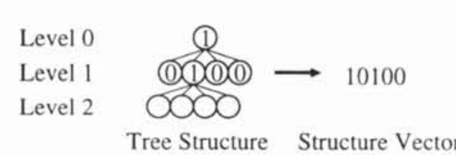

Figure 7: Extracting the structure vectors.

In the classification phase, test samples are classified according to their similarity with templates of textures. As the similarity measure, we use the following distance function

$$D(x,y) = D_e(x,y) \cdot \exp(D_s(x,y)/d) \tag{6}$$

where $D_e(x,y)$ and $D_s(x,y)$ stand for the Euclidean distance between energy maps and structure vectors, respectively. We note that the distance function $D(x,y)$ is reduced to $D_e(x,y)$ when x and y have the same tree-structure.

4 Experiments

4.1 Experimental Data

We used 25 textures, which are showed in Figure 8, from Brodatz's album [5]. Each image, which is of size 512 × 512 pixels, was scanned with 100 dpi resolution with 256 gray levels.

Two sampling schemes were used: the first one allow for overlap between the training and test sets. The second one does not allow for overlap and results in disjoint training and test sets. We randomly sampled 100 subimages of size 256 × 256 for training and test samples, respectively.

The size of the smallest subimage, which is necessary as a criterion for stopping further decompositions, should be found experimentally. If the decomposed subimage is too small, the energy value may vary widely from sample to sample so that the feature may not be stable [3]. We set the size of the smallest subimage to 32 × 32 pixels which corresponds to three level of decomposition for sample images of size 256 × 256. We set the value of the parameter C in Figure 1 to 0.3, as in [3] and used Daubechies 20 as the wavelet filter.

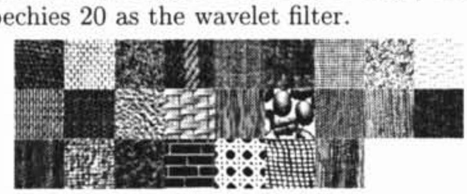

Figure 8: Textures for classification experiments.

4.2 Experimental Results

Table 1 shows the average of the classification rate of algorithms based on PSWT, $\mathrm{TSWT_{TD}}$ and $\mathrm{TSWT_{BU}}$ decomposition schemes. In Table 1, D-1 stands for the distance function given by equation (6), D-2 for the energy distance function D_e, and D-3 for $exp(D_s/d)$, respectively.

Table 1: Classification accuracy for each decomposition scheme.

Method	Sampling Scheme	Accuracy (%)		
		D-1	D-2	D-3
$\mathrm{TSWT_{BU}}$	Overlap	99.8	99.8	41.6
$\mathrm{TSWT_{TD}}$	Overlap	99.1	99.1	26.9
PSWT	Overlap	98.6	98.6	0.0
$\mathrm{TSWT_{BU}}$	No overlap	94.8	94.7	45.7
$\mathrm{TSWT_{TD}}$	No overlap	82.4	82.4	23.7
PSWT	No overlap	82.0	82.0	0.0

As can be seen from Table 1, $\mathrm{TSWT_{BU}}$ shows relatively higher classification accuracy than other two schemes, especially in the case of no overlapping between the training and test sets. Higher classification accuracy of $\mathrm{TSWT_{BU}}$, in terms of distance function D-3, points out the superiority in extracting structures of textures that are relatively more discriminative, as compared to the other two.

Table 2 shows the number of generated templates and the entropy of textures under PSWT, $\mathrm{TSWT_{TD}}$ and $\mathrm{TSWT_{BU}}$ decomposition schemes. We note that H_{local} and H are the local entropy and the entropy values averaged over 25 textures. They provide simple measures of the structural diversity of textures for a given decomposition scheme.

Table 2: Structural diversity of each decomposition scheme.

Method	Number of Templates	Structural Entropy		
		H_{local}	H_{global}	H
$\mathrm{TSWT_{BU}}$	2.44	0.68	3.76	0.18
$\mathrm{TSWT_{TD}}$	1.32	0.17	1.91	0.09
PSWT	1.00	0.00	0.00	0.00

The $\mathrm{TSWT_{BU}}$ showed larger structural diversity, as compared to $\mathrm{TSWT_{TD}}$, in terms of both the average number of templates and the average structural entropy. Noting that the former scheme shows higher classification accuracy, the aforementioned fact points out the superiority of the $\mathrm{TSWT_{BU}}$ scheme in generating relatively more representative and hence discriminative structures of textures.

5 Conclusion

In this paper, we provide an analysis of the structural stability of TSWT and proposed texture entropy as a metric to measure the structural stability of TSWT's. We also proposed a classification algorithm based on $\mathrm{TSWT_{BU}}$. The algorithm alleviated the problems associated with $\mathrm{TSWT_{TD}}$ based algorithm and demonstrated superior performance as compared to the algorithms based on PSWT and $\mathrm{TSWT_{TD}}$ decomposition schemes. Our current work includes incorporation of features other than energy to improve classification accuracy of the algorithm.

References

[1] T. R. Reed and J. M. H. du Buf, "A review of recent texture segmentation and feature extraction techniques," *CVGIM: Image Understanding*, vol. 57, no. 3, pp. 359–372, 1993.

[2] T. Randen and J. H. Husøy, "Filtering for texture classification: A comparative study," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 21, no. 4, pp. 291–310, 1999.

[3] T. Chang and C.-C. J. Kuo, "Texture analysis and classification with tree-structured wavelet transform," *IEEE Trans. Image Processing*, vol. 2, no. 40, pp. 429–441, 1993.

[4] R. R. Coifman and M. V. Wickerhauser, "Entropy-based algorithms for best basis selection," *IEEE Trans. Information Theory*, vol. 38, no. 2, pp. 713–718, 1992.

[5] P. Brodatz, *Textures: A Photographic Album for Artists & Designers*. New York: Dover, 1966.