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10—3 Facet Matching from an Uncalibrated Pair of Images

Lukas Theiler and Houda Chabbi *
Informatics Institute
University of Fribourg (Switzerland)

Abstract

Since for recognition tasks it is known that planar invariants are more easily obtained than others, decomposing a scene in terms of planar parts becomes very interresting. This paper presents a new approach to find the projections of planar surfaces in a pair of images. For this task we introduce *the facet* concept defined by linked edges (chains) and corners. We use collineations as projective information to match and verify their planarity. Our contribution consists in obtaining from an uncalibrated stereo pair of images a match of "planar" chains based on matched corners. Collineations are constrained by the fundamental matrix information and a Kalman filter approach is used to refine its computation.

Keywords: *Collineation, Chain Matching, Planarity, Feature Extraction*

1 Introduction

We are interested in *projective scene analysis.* Our aim is to develop a system which leads to a 3D projective reconstruction dedicated to 3D "planar" shape reconstruction of a scene observed from different viewpoints by an uncalibrated camera. This 3D reconstruction will provide a 3D projective recognition process. The only assumption is that the camera performs a pure perspective transformation and the scenes are made up several 3D object which can be decomposed in several planar components.

Several researchers work on the detection of planar information from images. Sinclair for instance looks for coplanar points in a scene [3] to locate a road. Hamid looks for coplanar lines [5] to determine coplanar configurations. In contrast to their work, our aim is not only to detect coplanar points or lines. We are looking for planar forms which correspond to a 3D surface of an object. Chabbi has already presented a projective approach to detect 2D facets in a triplet of images [2]. Our main difference to this work is that we no longer consider only polyhedral surfaces and introduce the use of collineation.

Our approach is only based on chains (linked edges) and corners extracted from two images. Using these two features we define in each image a *facet* f_i as a chain c_i with its set of nearest corners $\{p_i^k\}$. Our aim is to detect planar facets by using collineations. The collineation estimation is shown in section 2. The application of a Kalman filter to reestimate the collineation is shown in section 3. Results of our approach applied on pairs of images are presented in section 4.

2 Planar Facet Detection

First we match the corners in both images (I, I') and the fundamental matrix is calculated. Then for each facet f having at least three matched corners the collineation H is estimated to verify its planarity. This collineation H is a projective transformation related to the facet's projective plane $\pi : (n^T, d)$, defined up to a scale factor. Let us recall that given point $M \in \pi$, its two projections (m, m') are related by

$$\exists\lambda \neq 0, \quad \lambda m' = H.m \qquad (1)$$

This equation which is commonly used to calculate collineations has 8 parameters and can therefore be calculated with at least four matched corners. But since the fundamental matrix F and its epipoles (e, e') are known, we have $F = [e']_\times H$ [7] and then

$$H = M + e'\frac{n^T}{d} \quad \text{, where} \qquad (2)$$

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = -\frac{1}{\|e'\|^2}[e']_\times F$$

$$e'_\times = \begin{bmatrix} e'_x \\ e'_y \\ e'_z \end{bmatrix} = \begin{bmatrix} 0 & -e'_z & e'_y \\ e'_z & 0 & -e'_x \\ -e'_y & e'_x & 0 \end{bmatrix}$$

$$n = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

Therefore using (2) H has only 3 parameters, thus 3 matched corners are sufficient for its calculation.

*Address: Chemin du Musée 3, CH-1700 Fribourg Switzerland. E-mail: lukas.theiler,houda.chabbi@unifr.ch

Furthermore H is directly linked to the projective plane π, which is very useful for 3D projective reconstruction.

To match two facets, a confidence measurement $\mathcal{P}(f,f')$ is calculated between a pair of two facets $(f,f') \in I \times I'$ by projecting facet f's chain c using the collineation H, which is obtained by f's matched corners[6] :

$$\mathcal{P}(f,f') = \frac{\mathrm{Card}(H.c \cap c') + \mathrm{Card}(H^{-1}.c' \cap c)}{\mathrm{Card}(c) + \mathrm{Card}(c')} \quad (3)$$

$\mathcal{P}(f,f')$ takes into account the length of the chains ($\mathrm{Card}(c)$ and $\mathrm{Card}(c')$) and the amount of proximity of the projected chain $H.c$ to c' ($\mathrm{Card}(H.c \cap c')$). Finally we define f' as beeing the planar match of f by using the following criterion:

$$\mathcal{P}(f,f') = \max_{f'_k} \mathcal{P}(f,f'_k) > t, \quad (4)$$

where t is a given threshold

for all facets f'_k in image I' such that f and f'_k share a pair of matched corners. We usually take $t = 0.85$.

3 Collineation Reestimation using a Kalman Filter

To reduce the effect of noise and to improve the estimation of H of a planar facet match (f,f'), we poposed to reestimate the collination using a Kalman filter [1],[4]. The initial collineation is calculated by the corners of the facet f and the reestimation is performed using the facets chains. The advantage of this process is that we stop as soon as we completely fail to calculate H. Furthermore collineations between two coplanar facets can be updated very easily as soon as we detect another coplanar facet.

By making use of equation (1) and (2) [1] we obtain for a given pair of corresponding points $(P_i, P'_i) = ([u_i, v_i, 1]^T, [u'_i, v'_i, 1]^T)$ two linear dependent equations:

$$(-e'_z u'_i + e'_x) \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} u'_i m_{31} - m_{11} \\ u'_i m_{32} - m_{12} \\ u'_i m_{32} - m_{12} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \quad (5)$$

$$(-e'_z v'_i + e'_y) \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} v'_i m_{31} - m_{21} \\ v'_i m_{32} - m_{22} \\ v'_i m_{32} - m_{22} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \quad (6)$$

The dependency of these two equations comes from the epipolar constraint. While solving (5) minimizes the horizontal distance from a point to its epipolar line, (6) minimizes the vertical distance from a point to its epipolar line. Let us set up the Kalman equations for the horizontal case, since its very similar for the vertical case.

[1] assuming $||e'|| = 1$ and $d = -1$

For facet's set of chain pixels with coordinates $[u_i, v_i, 1]^T$ and their corresponding pixels $[u'_i, v'_i, 1]^T$ we can write (5) as

$$f(z_i, x) = \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} ((e_x - e_z u'_i) \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} + \begin{bmatrix} m_{11} - u'_i m_{31} \\ m_{12} - u'_i m_{32} \\ m_{12} - u'_i m_{32} \end{bmatrix}) \quad (7)$$

We need to minimize $f(z_i, x)$for the data vector $z_i = [u_i, v_i, u'_i]^T$ and the collineation's associated plane $x = [n_x, n_y, n_z]^T$, called state vector. The Kalman measurement equation is given by

$$y_i = M_i x_i + w_i \quad (8)$$

with

$$\begin{aligned} M_i &= \frac{\partial f(\hat{z}_i, x_{i-1})}{\partial x} \\ &= (e_x - e_z u') \begin{bmatrix} u_i \\ v_i \\ u'_i \end{bmatrix} \\ w_i &= \frac{\partial f(\hat{z}_i, x_{i-1})}{\partial z}(z_i - \hat{z}_i) \\ &= \begin{bmatrix} e_x n_x + m_{11} - u'_i(e_z n_x + m_{31}) \\ e_y n_y + m_{12} - u'_i(e_z n_y + m_{32}) \\ \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \begin{bmatrix} -e_z n_x - m_{31} \\ -e_z n_y - m_{32} \\ -e_z n_z - m_{33} \end{bmatrix} \end{bmatrix} \\ y_i &= -f(\hat{z}_i, x_{i-1}) - M_i \hat{x} \\ &= \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \begin{bmatrix} u'_i m_{31} - m_{11} \\ u'_i m_{32} - m_{12} \\ u'_i m_{32} - m_{12} \end{bmatrix} \end{aligned}$$

The state vector $x = [n_x, n_y, n_z]^T$ in equation (8) is now estimated by using a Kalman filter. For each new given couple of corresponding chain pixels we find a corrected estimate $\hat{x}$ by the following steps:

- Predict the measurement covariance $W_i = w_i \Lambda w_i^T$, where Λ is the error covariance of a pixel
- Calculate the Kalman gain $K_i = S_{i-1} M_i^T (W_i + M_i S_{i-1} M_i^T)^{-1}$
- Update the error covariance $S_i = (I - K_i M_i) S_{i-1}$
- Update the estimate with measurement y_i $\hat{x}_i = \hat{x}_{i-1} + K_i(y_i - M_i \hat{x}_{i-1})$

A crucial issue in the Kalman filter application is the initial data $\hat{x}_0$. Fortunately this is no problem in our application, since we can always calculate an initial collineation for a facet by using its corners . The collineation matrix is then updated by reestimating H for one couple of chain points after the other, choosen in random order. The interesting thing in this recursive process is that we can stop when the computation of H fails. Furthermore, the collineation between two facets can even be updated very easily as soon as we detect another coplanar facet.

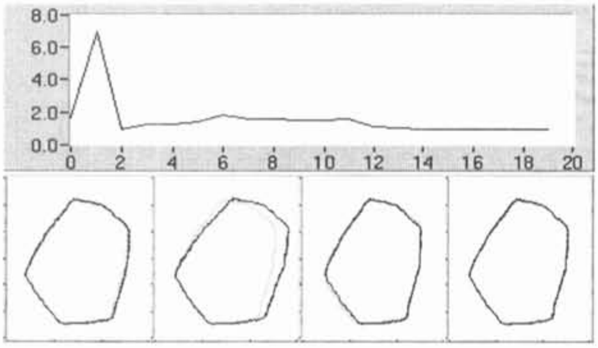

Figure 1: *Error development of the facet collineation using a Kalman filter.*

The top of Fig. 1 shows the error measured as the mean eucledian distances from the projected chain point to the points of the chain. The bottom of Fig.1 presents in gray the facet's chain c' and in black the projection $H.c$ shown at iteration 1,2,10 and 20. These images show the convergence to a better estimation of the facet collineation H. The initial divergence of the solution is a very typical development of Kalman filters.

4 Results

Let us now examine closely the results of the facet detection process by only regarding a pair of images. The following figures show the results of the facet matching process in detail. The first pair of images Fig. 2 shows all chains and corners that were found in two different views of the house.

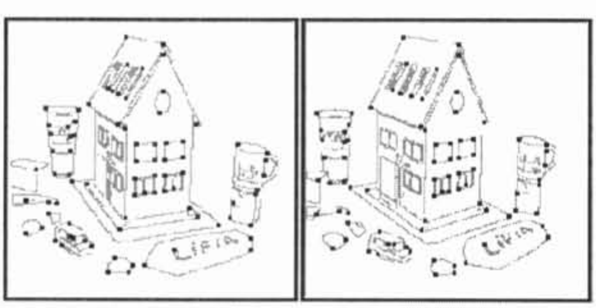

Figure 2: Chains and corners in two images

The first step is to find the projectable facets, those that have at least three corners. The pair of images shown in Fig. 3 show all projectable facets.

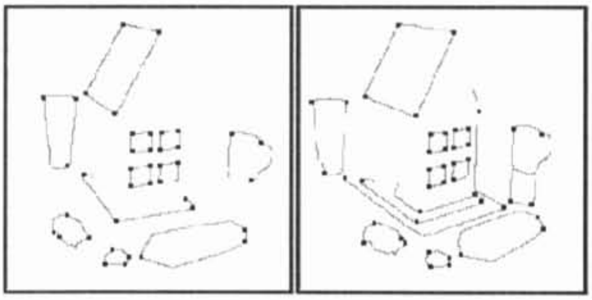

Figure 3: Projectable faces

We have set a threshold of $t = 0.85$ for the facet matching criterion (5). Unter that condition, 85 percent of the projection of a face must fall on its predicted facet. We found the facets in Fig. 4 pass the planarity test.

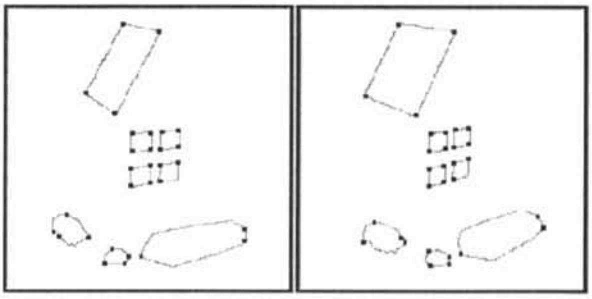

Figure 4: Planar 2D faces

If we look closely at the images, we see that the car on the left bottom in the images is recognized to be planar. Obviously cars are very rarely planar! But if we look at the edges of the car, we can easily imagine that the boundary of the car is very close to a planar cut of the car. Therefore the car is considered to be planar in this example. But nevertheless it is quite astonishing that we found a lot of planar 2D facets, even if we only estimate the chain collineation by their close corners.

Fig. 5[2] shows results of the planar facet matching process applied on three couples of images selected from a sequence of images. Depending on the pair of images, some planar facets could not be matched because they include less than three corners, for instance the conic on the front of the house.

[2]The graylevel images are from ftp://ftp.imag.fr/pub/MOVI/IMAGES/House

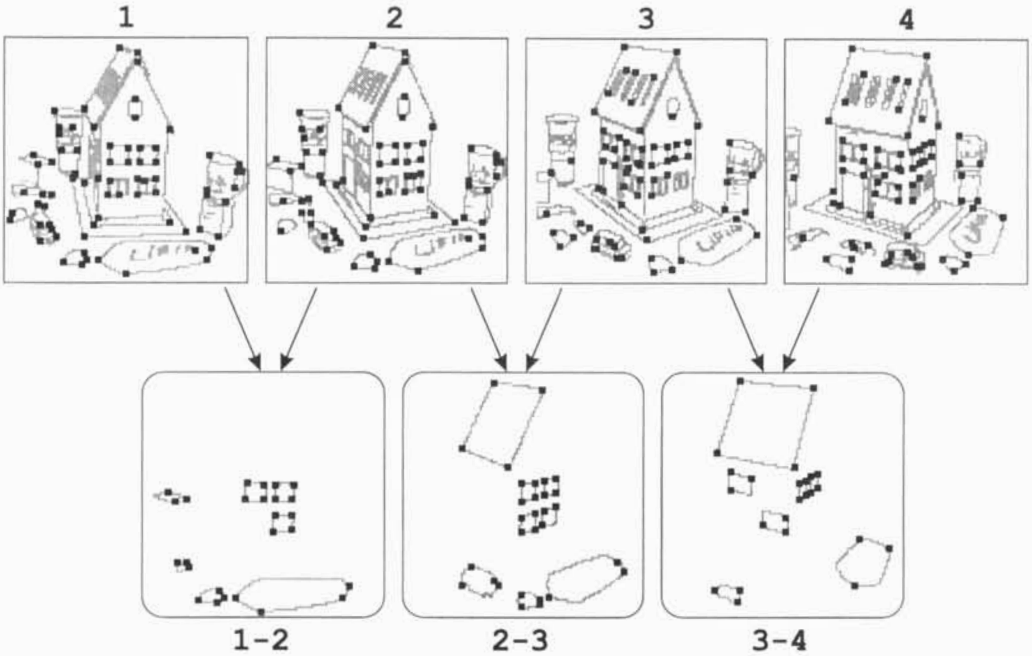

Figure 5: *Four views of the house and the detected planar facets by image pair.*

5 Conclusion

In this paper, we were looking to identify 3D object's planar surfaces by matching its 2D projections a in a pair of images. The approach is based on computing the collineation between two facets. Facets are features extracted from corners and linked edges. The facet matching process is initialized by its matched corners and the planarity is verified by taking its chain. The collineation between two facets is refined using a Kalman filter approach.

In summary, results are promising and we have several possibilities to enhance the number of matched facets. The most important concerns the extension of the image pair matching process to a sequence of images. This allows to complete views with problematic facets because these facets can be clearly identified as beeing planar in other views. We obtain a pertinent detection for problematic image pairs. In fact, the image sequence matching process can be seen as a planar facet *tracking* process.

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