MVA '98 IAPR Workshop on Machine Vision Applications, Nov. 17-19, 1998, Makuhari, Chiba, Japan

8—23

Cooperative Relaxation Algorithm of Range Images Using Surface Curvatures

Eung-Joo Lee [†], Ki-Ho Hyun [††], Gak Min*, Ki-Ryong Kwon**, and In-Gab Jeong***

[†] Dept. of I&C Eng., Tongmyong Univ. of Information Technology, [††] Dept. of Computer Eng., Youngsan Univ., *Taegu TV Technical Operations Division, KBS, ** Dept. of Electronic Eng., Pusan Univ. of Foreign Studies, *** Dept. of Electronics, Yecheon Junior College

Abstract

In this paper, we propose a cooperative algorithm between curvatures to escape local minima and choose a optimal possibility to preserve edge. The proposed algorithm between curvatures is relaxation in which weights of center pixel's and neighbor pixel's possibility are determined adaptively using deviation of the other curvature.

1. Introduction

Surface-based representations of range images appear to be most popular, and are generally constructed by defining primitives and relations between primitives, which naturally leads to object recognition by matching relational structures.

Surface curvature features, such as mean and Gaussian curvature, are invariant to rotations, translations, and changes in surface parameterization and therefore are desirable properties for 3D object recognition. But the effect of noise because the curvature computation process involves the estimation of the second partial derivatives which are highly sensitive to noise.

Many researches have been conducted, among them, Li used energy function through neural network to solve the noise problem[8,9] and recently a research based on pyramid structure to distinguish the range image was presented but it has the disadvantage of not being able to identify the boundary between the regions.

In this paper, to solve the noise problem without blurring, the surface segmentation is carried out by a minimizing energy function where the energy function is formulated from the constraints. Here, we intend to improve the energy function using surface curvature in two ways. First, we define a three competing constraints for energy function: preservation, similarity, uniqueness. Based on these constraints, we want to find the best possibility of its curvature sign so that each pixel in the image matches to the correct surface segmentation. Second, a relaxation algorithm cooperate with H and K curvature is to reduce the capability of converging the local minimum. If a energy function is using only one curvature, it is difficult to modified the initial misclassification due to noise.

Therefore, to solve the problem, we derive a relaxation algorithm having its minimum value if a deviation of the curvature is small in a window of the mean curvature, the neighbor points of the center are more weighted possibility in the same window of the Gaussian curvature, or if a deviation is large, the center point of the window is more weighted possibility. When the network is evolves, the experimental results are to protect the distortion of the boundary.

2. Surface Curvature

According to the fundamental existence and uniqueness theorem of surfaces from differential geometry, the local shape of an arbitary smooth surface is uniquely determined by the six parameters in the first

[†]Address: 535 Yongdandg-Dong, Nam-Gu, Pusan, Korea.
E-mail: ejlee@tmic.tit.ac.kr
[††]Address: hkh2170@chollian.dacom.co.kr

and second fundamental forms of the surface. The information contained in these six parameters can be properly expressed in terms of the mean curvature and Gaussian curvature as follows :

$$H = \frac{k1 + k2}{2}, \quad K = k1 \times k2 \qquad (1)$$

where, k_1 is the maximum principal curvature and k_2 is the minimum principal curvature. The signs of Gaussian curvature and mean curvature yield 8 possible surface types as shown in the Table 1.

Table 1. Surface types for the signs of curvatures.

H \ K	−	0	+
−	saddle ridge	ridge	peak
0	minimal	flat	✕
+	saddle valley	valley	pit

The most difficult problem in computing surface curvature from an image is, as widely reported, the effect of noise because the curvature computation process involves the estimation of the second partial derivatives which are highly sensitive to noise. However, improper smoothing such as over-smoothing across discontinuities often lead to the distortion of surface shapes, violating physical plausibility. These bring about a conflict between the smoothness required by the feature extraction procedures and the preservation of the local structure of the original surface desired by object recognition procedures. Here, we formulate the problem of invariant surface segmentation as that of finding minimal solution of energy functionals involving discontinuities.

3. Cooperative Relaxation Algorithm

The surface segmentation problem is defined in terms of finding a best result that satisfies three competing constraints such as preservation, similarity and uniqueness. Combining above three constraint, the total energy function is modeled as follows:

$$\begin{aligned} E = & -w_a \sum_i^{N_r} \sum_j^{N_c} \sum_k^{L} P_{(i,j,k)} \cdot v_{(i,j,k)} \\ & - w_b \sum_i^{N_r} \sum_j^{N_c} \sum_k^{L} \sum_{s \in S} v_{(i,j,k)} \cdot v_{((i,j)+s,k)} \\ & + w_c \sum_i^{N_r} \sum_j^{N_c} \sum_k^{L} v_{(i,j,k)} \Big(\sum_{n \neq k}^{L} v_{(i,j,k)} + 1 \Big) \end{aligned} \qquad (2)$$

where $P(i,j,k)$ is the initial input of node. The first term of energy function is determined by the possibility in proportion of sign k using $v_{(i,j,k)}$ at point (i,j). The second term is formulated to have its minimum value in the case that, if the sign of a node is k, then the sign of its neighboring point is k. The third term is formulated to have its minimum when only one node is 1 and the rest, $v_{(i,j,n)}$ $n \neq k$, are -1.

By comparing the terms in (2) with the Hopfield energy function, we can determine the interconnection strengths and bias inputs as follows:

$$T(i,j,k{:}l,m,n) = 2 w_b \cdot \sum_{s \in S} \delta((i,j),(l,m)+s) \cdot \delta(k,n) - 2 w_c \cdot \sum_n \delta((i,j),(l,m)) \cdot (1 - \delta(k,n)), \qquad (3)$$

$$I(i,j,k) = w_a \cdot P(i,j,k) - w_c \qquad (4)$$

where $\delta(i,j)$ is the Dirac delta function.

The segmentation here is formulated as a minimization problem, finding a minimal solution that minimizes appropriate energy functional. To get this solution, we defined a possibility which measures the cost arising from every individual unary mapping from curvature value to the sign. The possibility embodies more a priori knowledge about the relationship between a curvature value and its sign. Moreover, the elaborate possibility can be devised for the segmentation purpose.

$$v^{(0)}_{(i,j,-)} = \begin{cases} -2\exp(-\lambda^2(i,j)/\tau_-) + 1 & \lambda(i,j) < 0 \\ -1 & \lambda(i,j) \geq 0, \end{cases}$$

$$v^{(0)}_{(i,j,0)} = \begin{cases} 2\exp(-\lambda^2(i,j)/\tau_-) - 1 & \lambda(i,j) < 0 \\ 2\exp(-\lambda^2(i,j)/\tau_+) - 1 & \lambda(i,j) \geq 0, \end{cases} \qquad (5)$$

$$v^{(0)}_{(i,j,+)} = \begin{cases} -1 & \lambda(i,j) < 0 \\ -2\exp(-\lambda^2(i,j)/\tau_+) + 1 & \lambda(i,j) \geq 0, \end{cases}$$

where $\lambda(i,j)$ is the H or K curvature value of the point (i,j), and τ_-, and τ_+ is $-\frac{T_-^2}{\ln(0.5)}$ and $-\frac{T_+^2}{\ln(0.5)}$, respectively. The initial input value of node is between -1 and 1 as follows:

$$P(i,j,k) = v^{(0)}_{(i,j,k)} \tag{6}$$

where k is the signs of mean and Gaussian curvature at a point (i,j), and $v^{(0)}_{(i,j,k)}$ is the possibility of k sign which is initially classified the a point using curvature.

In each model of the relaxation scheme, the net input $u(i,j,k)$ for node (i,j,k) is given by the sum of $T(i,j,k{:}l,m,n)\cdot v(i,j,k)$ and the bias input $I(i,j,k)$, which is random and asynchronous. The next output of node is given by

$$u^{(t+1)}_{(i,j,k)} = (w_a P(ij,k) - w_c) + w_b \sum_{s \in S} v^{(t)}((i,j)+s,k) - w_c \sum_{n \neq k}^{L} v^{(t)}_{(i,j,n)} \tag{7}$$

The allowable output value of each node is between -1 and 1 which is calculated by the sigmoid function as follows:

$$v^{(t+1)}_{(i,j,k)} = \tanh\left(\frac{u^{(t+1)}_{(i,j,k)}}{u0}\right) \tag{8}$$

However, if we consider the only one curvature information to iterate the energy function, the problem is difficult to modify the possibility with its neighbor value when the initial possibility is too large due to noise effect. It means whether the energy decrease, the solution is easy to converges the local minimum of energy surface.

Another problem with the conventional method is that of oversmoothing. A relaxation algorithm cooperate whith H and K curvature is to reduce the capability of converging the local minimum. Also, the result is preserved boundary because of finding the best possibility near smoothed discontinuities.

The updating rule ensures that the minimization process always converges a stable state after a sufficient number of steps because the updating is always performed in such a way that it reduces a local energy. The result of each iteration is used as the initial configuration for the next. The process repeats until it converges. The cooperative function is include the link between H and K. If the deviation of curvature is small in a window of the mean curvature, it has more capability the region in K curvature will be the same sign. It is easy to escape from the local minima as the iteration increases. The link between curvature uses the standard deviation and the node output of each curvature is giben by as follows:

$$v^{(t+1)}_{H(i,j,k)} = \sigma K(i,j,k) \cdot v^{(t)}_{H(i,j,k)} + (1-\sigma K(i,j,k)) \cdot \overline{v^{(t)}_{H(i,j,k)}}$$

(9)

$$v^{(t+1)}_{K(i,j,k)} = \sigma H(i,j,k) \cdot v^{(t)}_{K(i,j,k)} + (1-\sigma H(i,j,k)) \cdot \overline{v^{(t)}_{K(i,j,k)}}$$

where $\sigma K(i,j,k)$ and $\sigma H(i,j,k)$ are the standard deviations of the node k in the window of the H and K, respectively. $\overline{v^{(t)}_{H(i,j,k)}}$ and $\overline{v^{(t)}_{K(i,j,k)}}$ are the average of the node k in the same window. Fig.1 shows the cooperative relaxation model of H and K. In this model, the next output of a point in K model determines the relative weight between the neighbor output and the output itself from the deviation. The interaction model between H and K is to ensure the convergence to stable minima when the relaxation process proceeds with the deviation of the possibility.

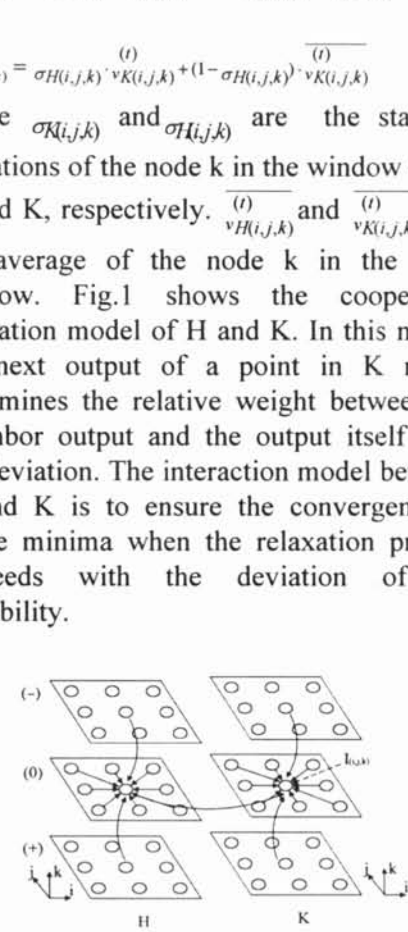


Fig.1. The cooperative relaxation of K and H

$$\begin{cases} v^{(t+1)}_{H(i,j,k)} = v^{(t+1)}_{H(i+u,j+v,k)} \\ v^{(t+1)}_{K(i,j,k)} = v^{(t+1)}_{H(i+u,j+v,k)} \end{cases} \tag{10}$$

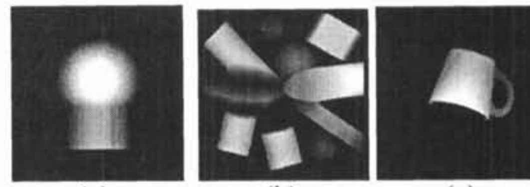
While the iteration process is goes on, its result is appears to preserve the boundary near the discontinuities. After the iteration process, the output of the node, $v_{(i,j,k)}$ has continuous value between -1 and 1. And the sign of the node is determined from

$$h_{(i,j)}^{(t+1)} = \begin{cases} - & v_{(i,j,-)}^{(t+1)} > 0 \\ 0 & v_{(i,j,0)}^{(t+1)} > 0 \\ + & v_{(i,j,+)}^{(t+1)} > 0 \end{cases} \quad (11)$$

The iteration process continues until $h_{(i,j)}^{(t+1)} = h_{(i,j)}^{(t)}$ if no change happens and the combination of the result can classify 8 different surface types. After this, H-K sign based surface type classification can be obtained according to the combinations of the H sign and the K sign.

4. Experimental Results

In the experiments, we use two real data and a synthesized imageas shown in Fig. 2. In Fig. 2(a), the bulb image which is combined with pit and ridge has Gaussian noise of $\sigma=2$ added. The real images are cup and block data from NRCC(National Research Council Canada). The synthesized and real images are of size 128×128 and 256×256, respectively. The threshold values of zero sign for initial segmentation are $H = \pm 0.005$ and $K = \pm 0.0001$ for bulb and block, $H = \pm 0.002$ and $K = \pm 0.0005$ for cup image. Three weights for the relative importance of the energy function are $w_a=1$, $w_b=0.5$, $w_c=0.5$ and the threshold for the correction of possibility near boundary is 0.5. Fig. 3 shows the segmentation result of separate relaxation model of H-K is converged the local minima which is cause from the noise. The classification of regions between pit and ridge are not as boundaries as saddle valley and some small regions are misclassified. Fig. 3 shows the results of separative relaxation algorithm and Fig. 4 shows the results of cooperative relaxation algorithm. As shown in Fig. 4, the proposed algorithm segment and classifies the range image more accurately compared to the separative relaxation algorithm.



(a) (b) (c)

Fig. 2. Input range image.
(a) bulb; (b) block; (c) cup image.

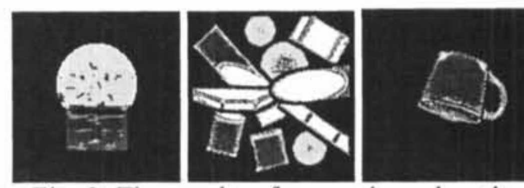


Fig. 3. The results of seperative relaxation

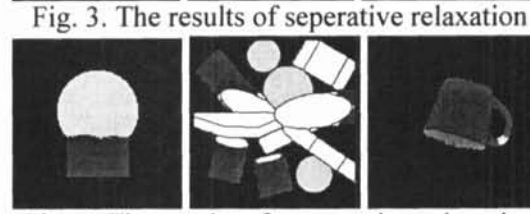


Fig. 4. The results of cooperative relaxation

5. Conclusions

In this paper, we define a 3-D object segmentation and classification scheme based on cooperative relaxation method for the escape of local minima. The proposed algorithm preserve the boundary near the discontinuities as well as reduce the noise effect. In the experiments, the proposed algorithm segment and classifies the range image more accurately compared to the separative relaxation algorithm.

References

[1] A. Mitiche and J.K. Aggarwal, "Detection of edges using range information," IEEE Trans. on PAMI, vol. 5 no.2, pp. 174-178, 1983.

[2] P. J. Besel and R. C. Jain, "Segmentation through symbolic surface descriptions," IEEE, pp. 77-85, 1986.

[3] N. Yokoya and M. D. Levine,"Range image segmentation based on differential geometry:a hybrid approach," IEEE Trans. on PAMI, vol. 11, no. 6, pp. 643-649, 1989.

[4] S. Z. Li,"Invariant surface segmentation through energy minimization with discontinuities," IJCV, vol.5:2, pp.161-194, 1990.