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A Computed Imaging System Using Wavelets Sampling Model

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Abstract

A computed 3D imaging system based on local image reconstruction (LIR) using Wavelets Sampling Model is presented. In this paper, by means of combination of wavelets sampling model and singular value decomposition, moderate CT image with least squares of error can be reconstructed from a limited number of projections by only once operation of matrix multiplication. As a result, local image reconstruction idea is proposed to reduce quantity of calculation and memory. In addition, as this algorithm is suitable for being achieved by massive parallel processing, we show an outline of VLSI parallel processor with it. As the method reduces reconstructing computation and simplifies projection data acquisition, it becomes possible to develop faster and more compact industry-oriented 3D CT system. Computer simulation of 3D image and design for VLSI and industry-oriented 3D CT system with this method are presented at last.

1 Introduction

In many industrial application of computed tomography (CT) other than medical diagnostic areas, there are circumstances that a sufficient amount of projection data can't be easily obtained. For instance, in cyclotron beam density measurements and nondestructive diagnostics on belt conveyer, in order to ensure efficiency, projection data had to be collected over a few views. In such limited-view cases, if using a standard multi-data reconstruction algorithm, such as the filtered back-projection (FBP) method, results generally are image reconstruction deteriorated with severe artifacts. In industry application, furthermore, faster and cheaper CT equipment become more important and necessary because components must be nondestructive inspections on belt conveyer. Because of the important of the limited-views problem, many specialized algorithms, which have attained some degree of success, have been introduced over the past decades [1]-[2]. However, blurring is still one of the major factors that degrade the resolution and produce shape distortion.

In our previous work, a method called fast model reconstruction (FMR), which obtains reconstruction image by using both a truncated singular value decomposition (truncated SVD) and sampling model object, have been proposed [3]-[5]. With FMR, even if reconstructing from fewer projection data with less information, because effect from whole of the projection paths is contained in the model, results have better redundancy. It is found that the implementation of fast image reconstruction can be made only with matrix calculation.

In previous work, classical Shannon's sampling function (sinc) always be use as the distributed function. The sinc function is an ideal filter. However, because it is an infinite function, there is contradictory between reconstruction efficiency and error controlling.

In the past decade, wavelet theorem has been introduced into signal process. It is proposed that the classical Shannon sampling theorem can be extended to the subspaces used in the multi-resolution analysis in wavelet theory [10].

In this paper, we introduce wavelet theorem into sampling model image reconstruction method. We present that wavelet function can take the place of classical Shannon's function used in sampling model object. Suitable wavelets basis is chosen to construct wavelet sampling function. The sampling model reconstruction method with the wavelet sampling function has good computational efficiency with less reconstruction error. In order to increase computation speed, we use local area reconstruction method replace our previous method, this method solves the problem by calculation a great deal of small blocks of sub-matrices other than a big matrix. We present that the algorithm of this method is suitable for VLSI parallel processing.

2 Image Reconstruction Model on Shannon's Sampling Function

In this paper, we make two fundamental assumptions at first. The first is that original continuous signal $g(x,y)$ is band-limited with a cut

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frequency ω_m. The other is that the continuous signal g is sampled at or above the Nyquist frequency--$2\omega_m$. The Shannon's sampling theorem states that any band-limited continuous signal g, is sampled at or above its Nyquist frequency (yielding the discrete function $g(x_k,y_i)$), can be completely reconstructed. To a 2-D object $g(x,y)$, its Radon transform (projection data) be represented as a line integration of intensity g(x,y) along each ray:

$$p = \int_0^L \sum_{k=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} g(x_k, y_i) \cdot sinc(x - x_k) \cdot sinc(y - y_i)\, dl \approx \sum_0^u \sum_0^u g(x_k, y_i) \cdot C_{k,i} \quad (1)$$

where $x_k=kT$, $y_i=iT$, H is the length of ray from entering point to escaping point through projection interval, h is the length from entering point to any point on the ray.

The $c_{k,j}$, in this formula, is called "Line Integration Effect Coefficient (LIEC)". It only depends on located relation between rays and sample points, and has no relationship with density of sample points. It expresses the contribution of sample points density to ray projection value. By means of (1) of every rays, we have projection matrix as:

$$P = C \cdot g + e \quad (2)$$

where $P=\{p_m\}$ and $e=\{e_m\}$ are m-D vector of projection data and error. $g=\{g_n\}$ is n-D vector of sampled-value. $C=\{c_{mn}\}$ is m×n matrix. The element of matrix $c_{mn}=C_{k,i}$ is the LIEC which shows how much contribution of nth sampled-value to mth ray.

Because matrix C depends on methods of sampling and projecting, it is called "Sampling Projection Model Matrix". Using singular value decomposition and minimum norm least square solution of C in (2), it is easy to see that reconstruction equation be reduced to:

$$g = V\Lambda^{-}U^{t}P = C^{+}P \quad (3)$$

In advance, we calculate m×n constant matrix C^+, and then by just once multiplying with measured projection data P (m-D vector) , we can easily obtain n-D vector sampling point value g. As a result, we investigate an image reconstruction method on sampling model object.

3 Wavelet Sampling Model Object

The ideal reconstruction process with sinc function, although realizable, because sinc is infinite and decays slowly, in order to reconstruct an image from its sample dataset perfectly, requires a integration from an infinite sum, so is not practical. Therefore, a certain integration area must be determined to approximate to an ideal low-pass filter, common solutions are useing a truncated sinc or a window function. Unfortunately, notable truncation error (blurring, aliasing, and ringing) and non-sinc error arise with these methods.

In this section, we substitute wavelets sampling theorem for the Shannon's sampling theorem to set up sampling model object. With wavelet sampling model object, the reconstruction error is remarkably decreased and calculation efficiency is increased.

At first, we propose that there is a sampling theorem imbedded in any wavelet theory. That is, if we begin with any scaling function satisfying the required properties in [8], there is a natural sampling function $S_n(t)$ that gives a sampling expansion for $f \in V_0$.

We suppose $\phi(t)$ is a real continuous scaling function, and whose translates and scales $\{\phi(2^j x-k)\}$ form an orthonormal basis of a subspace V_j of $L^2(R)$. While both a continuous signal $f(t)$ and $\phi(t) \in V_0$, we have the wavelet transform of f as:

$$f(n) = \sum_k a_k \phi(n-k.) \quad (4)$$

and with it, the Fourier transform of discrete function $f(n)$ and continuous function $f(t)$ can be written as:

$$\hat{f}^{*}(\omega) = \sum_n \sum_k a_k \phi(n-k) e^{-jn\omega} = \hat{a}^{*}(\omega) \cdot \hat{\phi}^{*}(\omega) \quad (5)$$

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} \sum a_k \phi(n-k) e^{-jt\omega} dx = \hat{a}^{*}(\omega) \cdot \hat{\phi}(\omega) \quad (6)$$

where $\hat{a}^{*}$ is in $L^2(0,1)$ and $\hat{\phi}^{*}(\omega)$ is continuous.

From (5) and (6), the wavelet sampling function is obtained and shown as:

$$S(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\hat{\phi}(\omega)}{\hat{\phi}^{*}(\omega)} \cdot e^{j\omega t} dx \quad (7)$$

In the last ten years, several orthonormal wavelet basis for $L^2(R)$ have been constructed which share the best features of the Haar basis and the Littlewood-Paley basis, these new constructions have excellent localization properties in both time and frequency [8]. One of them is the Mayer basis, in which Fourier transform of its ϕ and ψ is compactly supported. We use Mayer wavelet to construct our wavelet sampling function S_m. According to the define of Mayer wavelet [9], S_m (Fig.1) can be obtained by (7):

$$S_m(t) = \frac{\sin \frac{2\pi}{3} \cdot t}{\pi t} + \frac{1}{\pi} \int_{2\pi/3}^{4\pi/3} S_m'(\omega) \cdot \cos\omega t \, dt \quad (8)$$

where

$$\hat{S}'_m(\omega) = \frac{\cos[\frac{\pi}{2}\upsilon(2-\frac{3}{2\pi}\omega)]}{\cos[\frac{\pi}{2}\upsilon(2-\frac{3}{2\pi}\omega)]+\cos[\frac{\pi}{2}\cdot\upsilon(\frac{3}{2\pi}\omega-1)]}$$

Although S_m is infinitely supported, it decays far more rapidly than sinc. We replace sinc with S_m to set up sampling projection model matrix C.

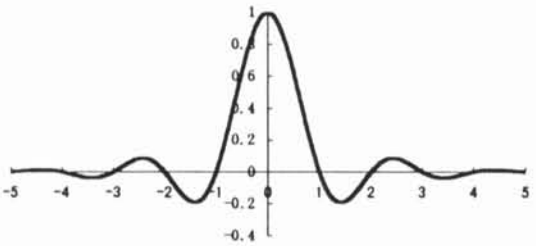


Fig.1 1-D Meyer sampling function Sm(t)

4 local area reconstruction method

Our previous method in section 2 is a method called whole area reconstruction (WAR) method because reconstruction of each sampling point uses all of the projections through whole reconstruction area. The reconstruction is fulfilled by only once multiplication of a big matrix.

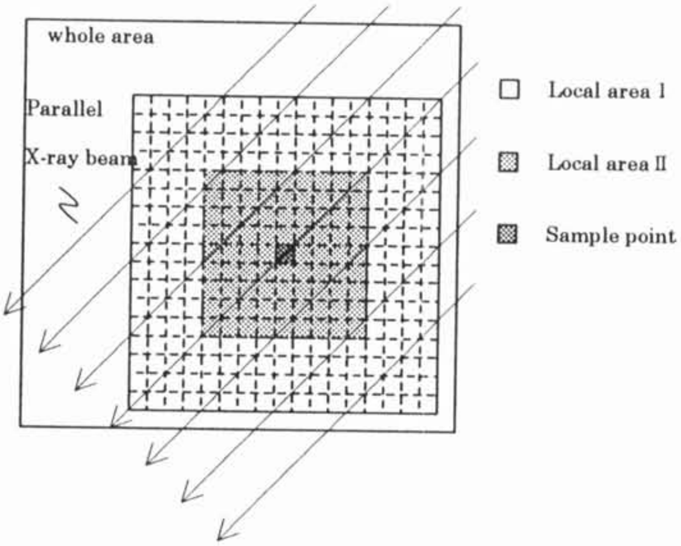
In theory, effect area of one sampling point is infinite, and in order to reconstruct a sampling point density, projections of infinite area is necessary. In practice in our previous work, all of the projections through projection area (whole area) other than through an infinite area were used to complete reconstruction. The projection area is still too big for a big size 3D object. In fact, notable effect area of a sampling point is much smaller than projection area generally. We call this effect area 'local area'. Most of information of sampling point density is contained in projections through this local area. Density of one sampling point can be reconstruction only by these projections, so we call our method local area reconstruction (LAR) method. It is obvious that much error (local area error) will occur in this method, however it is reduced by using wavelet model.

Fig.2(a) shows the relations among sampling point, local areas and whole area (We use parallel beam here). As examples, local area I and local area II are set as 9×9 and 17×17 points respectively. Fig.2(b) (area I) and Fig.2 (c) (area II) show that, with different effect area, one sampling point has different LIECs because there are different number of beam between the two areas, however the coefficient distribution are same. From them, it is obtained that C^+ (see equation (2)) also have same forms. And as a result, image reconstruction using two different local areas is same in the main. A theorem is drawn from these:

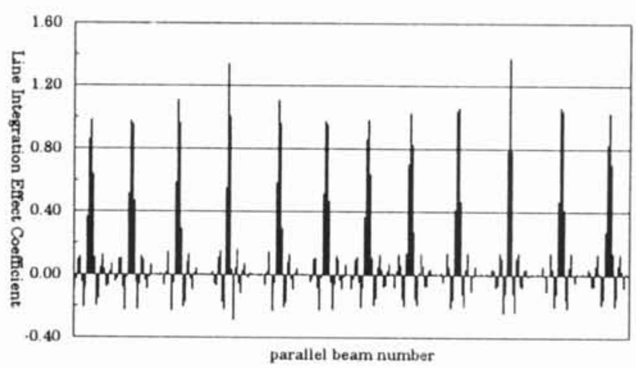
***Theorem 1** (local image reconstruction theorem):*

In sampling model space, generalized-inverse matrix of whole area LIECs can be obtained by means of generalized-inverse matrix of local area LIECs.

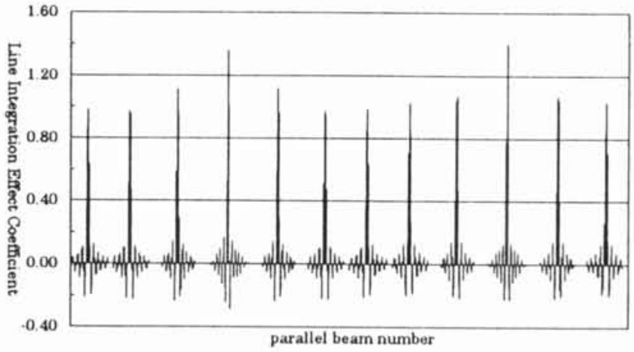
The theorem means that, image reconstruction can be complete either using in one time or one point by one point. But with the last one, computation and memory required can be reduced because big block matrix is solved using small blocks of sub-matrices.



(a) Projection relations



(b) Local area I



(c) local area II

Fig.2 Projection relations and LIECs with local area

5 Design for VLSI and 3D CT System

Because the algorithm of this method included multiplication and addition only, it is suitable for parallel processor by VLSI. We design our leased VLSI with the algorithm. In one chip, there are 32 calculation units (containing 300,000 gates, using 0.35μ rule). One VLSI chip be used to calculate reconstruction of one slice containing 512×512 sampling points in 3D object.

Using this VLSI, a VLSI parallel processor is designed. This parallel processor contains one control board and seven parallel arithmetic boards connected by PCI bus. There are 32 VLSI chips in one parallel arithmetic board, and all of the parallel arithmetic boards complete reconstruction calculation of 224 (32×7) slices under the control of control board.

A slanting rotate scan device is also presented. In this scanner, X-ray emitter (using parallel X-ray beam) rotates around inspected object with a 45° dip, X-ray detector obtaining projection data is under the inspected object. There are same LIEC matrices to every sampling point in each slice if letting inter-space of adjacent beams equal adjacent sampling points. Therefor, all of the sampling points (It is $512 \times 512 \times 224$ in our design) can be reconstruction in the same time.

With these devices, an industry-oriented FMR 3D CT system using 7,168 parallel computing units shown in Fig.3 is designed. At present, the system is implementig.

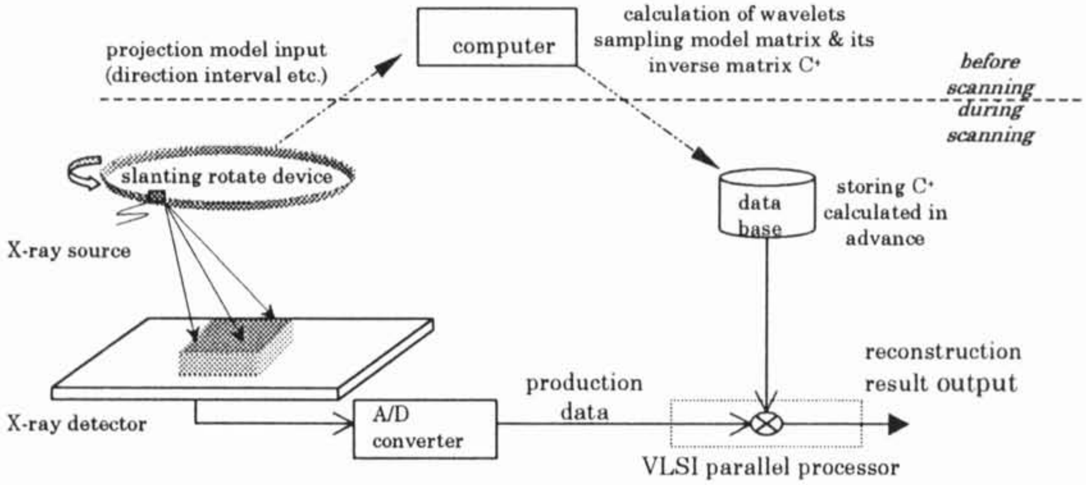


Fig.3 A structure of the 3D CT system

6 Conclusions

In this paper, we present a fast computed 3D imaging method based on local image reconstruction using wavelets sampling model. By using this method, a new type of industry-oriented CT system can reconstruct moderate CT image with least squares of error from only a few projection data. The method does not require a large amount of calculation after acquisition of projection data, and not require a large of memory for reconstruction. CT image can be obtained by only once operation of matrix multiplication. The structure of reconstructing computation is suitable for VLSI circuit by ASIC. As the method does not require a large amount of calculation and memory for reconstruction, and does not require a complicated mechanism for projection data acquisition, there is possibility of developing more compact industry-oriented 3D CT system.

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