

A New Approach for Modelisation of Ellipsoidal-based Shape.

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Abstract

In this paper, we propose a new method to detect ellipsoidal shapes. This method combine two well known approach : the Modified Hough Transform and the Elliptic Fourier Decomposition. The applications of this method are, for instance, medical imaging (ultrasound imaging, X-rayed, ...), biological imaging (cell count) or intelligent vision in robotic. We have apply our method to the particular problem of cranial outline of foetus in ultrasound imaging.

Keywords : ultrasound, 3D reconstruction, deformable model, cranial contour modelisation, Hough Transform, Fourier decomposition

1 INTRODUCTION

1.1 Current methods of ellipsoidal contour detection

Existing method of ellipses detection HT¹ based, are not able to work with thick outlines or gray scale images. Fit methods to thin outline can be sort as algebraic criteria method based ([5] [6]) and geometrical criteria method based ([4] [9] [8] [1] [7]). Use of HT, in the case of ellipses, leads to 5 dimensions hyperspace because this shape needs 5 parameters (center, two axes and direction). Direct application of HT leads to a number of calculations proportional to $C_5^n = \frac{n!}{5 \cdot (n-5)!}$ where n is the number of image points. Existing methods try to reduce the number of dimensions. In most cases, authors propose tests to detect ellipse center followed by the search of the other parameters. In our application, the outline, a cranial foetal contour, is ellipsoidal. So, we propose *a search of center and then a search of candidate dots which belong to the ellipse* followed by a modelisation describe in [2]. The originality

of our method is linked to this points : *outline is thick and is not a perfect ellipse, furthermore the image is noisy.*

1.2 Centre Search by Hough Transform (CSHT)

1.2.1 Existing methods

To perform an exact modeling of ellipsoidal outline with the method describe in [2], we must do a selection of dots belong to this outline. There is several methods to do this :

1. Computation, for each pair of points $M_1(x_1, y_1)$ and $M_2(x_2, y_2)$, of the center $O = (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ [7]. (duration proportional to C_n^2).
2. Previous method can be speed-up with the use of points which have the same gradient vector like in [7] and [8].
3. Use of horizontal and vertical scanning to produce two straight lines of symmetry. The intersection of this two lines is the center of ellipse [4].
4. A method describe in [9] is based on the notion of *pôle* and *polaire* describe in [3]. This method can be used to detect multiple ellipses.

1.2.2 Remarks and conclusion

The observation of HT results applied to a binary image leads to following remarks :

- Effect is *average filtering* : in fact, the HT performs a summation over the set of image points. So, we can expect a improvement of SNR².
- In the case of a only ellipsoidal shape in the image, the HT must gives only one maximum. The question is : is-it necessary to wait for the end of iterations to detect this maximum ?

So, to increase the speed, we propose two ways :

²Signal-to-Noise Ratio

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¹Hough Transform

- *Research of maximum for each iteration.*
Restriction could be the decrease in speed.
- *Decrease the number of starting points with a undersampled*

2 METHOD

2.1 Conventional method of CSHT

2.1.1 Introduction

For each pair of points, we calculate their middle and increase a counter inside the transformed image. Dimensions of transformed image are the same of origin image and the gray level of a point represent a "vote" to this point [7]. We have performed, with some test images, measurement of speed convergence of CSHT. We extract, from the image, a set I which represent *index*, i.e. the number of a pixel inside the image. (see section 2.1.2).

2.1.2 Example

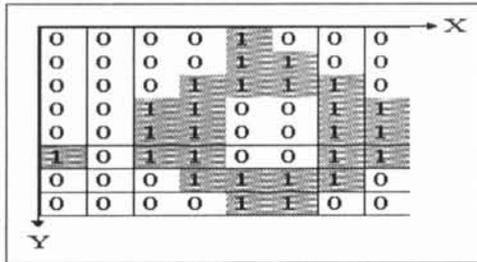


Figure 1: Test image

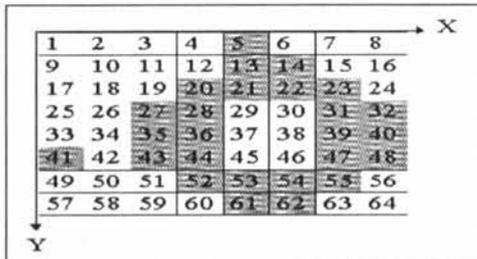


Figure 2: Index of test image

The test image is on figure 1. The set of image index, figure 2, is :

$$I = \{5, 13, 14, 20, 21, 22, 23, 27, 28, 31, 32, 35, 36, 39, 40, 41, 43, 44, 47, 48, 52, 53, 54, 55, 61, 62\} \quad (1)$$

We apply the CSHT to this set : we obtain the **index 38** and the transformed image is figure 3. The figure 4 represent the evolution of center index according to the iterations.

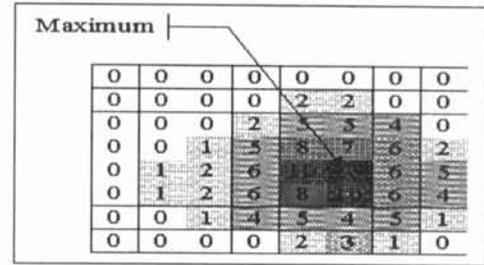


Figure 3: CSHT of test image

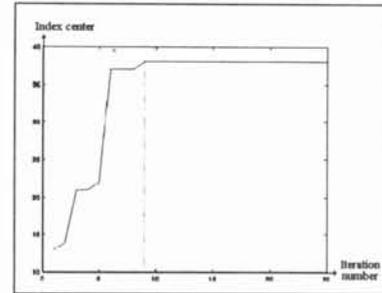


Figure 4: Index center evolution

2.2 Suggested method to increase calculation speed

We divide the set I into four subsets I_1, I_2, I_3 and I_4 such as $I_1 \cap I_2 \cap I_3 \cap I_4 = \emptyset$ and $I_1 \cup I_2 \cup I_3 \cup I_4 = I$ with :

$$\begin{cases} I_1 = \{5, 21, 23, 35, 53, 55\} \\ I_2 = \{20, 22, 36, 40, 52, 54\} \\ I_3 = \{13, 37, 31, 41, 43, 47, 61\} \\ I_4 = \{14, 28, 32, 44, 48, 62\} \end{cases} \quad (2)$$

This operation could be compare to a *undersampled* with a half frequency. We use one point in two in the two directions x and y (first order). We apply the algorithm of CSHT to this new layout : we obtain an index 37 from the fifth iteration, which represent only one pixel error (see figure 5). So, the speed of the algorithm is increased. For a second order in x and y directions, we obtain approximately the same speed as previous.

2.3 CSHT algorithms

2.3.1 Improvement of conventional algorithm

On figure 6, I represent the set of index and $N = \text{card}(I)$. Each arrow represent a half summation between two elements i and $(i+k+1)$ with i from 1 to $(N-1)$ and k from $(i+1)$ to N . So, the calculation number is : $C_2^N = \frac{N!}{2 \cdot (n-2)!}$. The first improvement we propose is **the observation of convergence by the measurement of center index value for each iteration**

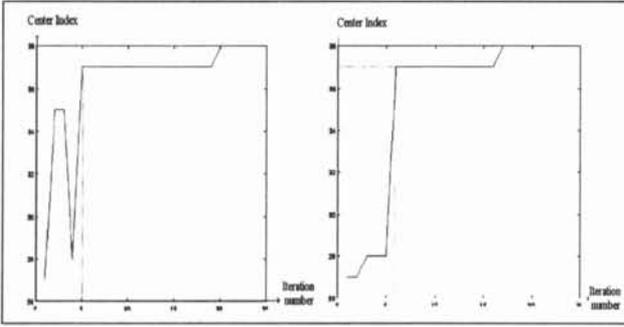


Figure 5: Evolution of index center (first and second order in x and y)

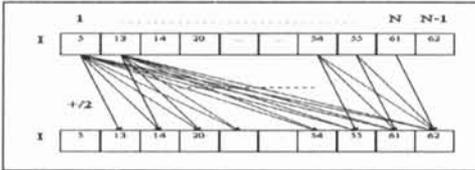


Figure 6: Conventional algorithm [7]

2.3.2 Improvement of conventional algorithm by undersampled

- **First stage :** The algorithm is showed on figure 7 where $i = 1 \dots M$ with M , number of subsets and $N_i = \text{card}(I_i)$. This stage is applied to each subset I_i . For each iteration, convergence is measured.
- **Second stage :** The algorithm is showed on figure 8. This stage is applied between two different subsets in case the previous stage doesn't converge.

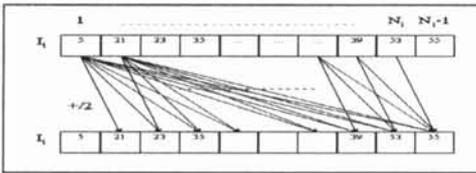


Figure 7: First stage of undersampled algorithm. Here, we have $i = 1$

2.4 Convergence measurement criterion

If we want a efficient CSHT algorithm, it's necessary to use a criterion which allows to stop it when the center index is reached for a certainty. However, the speed of convergence is linked to some parameters :

- **Number of points to compute.** The more this number is high, the longer convergence slow down.

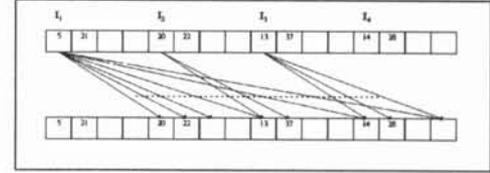


Figure 8: Second stage of undersampled algorithm

- **Undersampled.** The more this order is high, the longer convergence speed up. However, the inaccuracy is increased.
- **Delay to hit convergence.** This parameter is linked to SNR.

So, we have used a TCE^3 named τ : if during the TCE the center index is unchanged, then the algorithm is stopped. The TCE value τ will be adjust according to the image features. On figure 10, we can see convergence evolution of figure 9 center with $\tau = 500$.



Figure 9: Test image to the TCE comparison

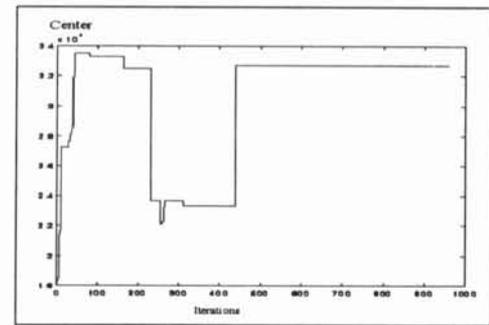


Figure 10: TCE $\tau = 500$

2.5 Candidate points research CPR

We use a straightforward algorithm which detect points belong to outline in a $Q * Q$ neighbourhood around the center. For $Q = 1$, we obtain a **selective filter**. When Q increase, selectivity decrease.

2.6 Signal Noise Ratio rising

Combination of CSHT and CPR lead to a non-linear filtering which allows to **increase the SNR**.

³time constant equivalent

However, this filter is limited to symmetrical shapes.

3 RESULTS

3.1 Test image

To test the method, we use an ultrasound scan image (figure 11) of a foetal cranial outline. It's a 320*240 pixels picture with a 256 gray-scale. We use an algorithm described in [2]. We obtain a binary outline with noise (figure 11).

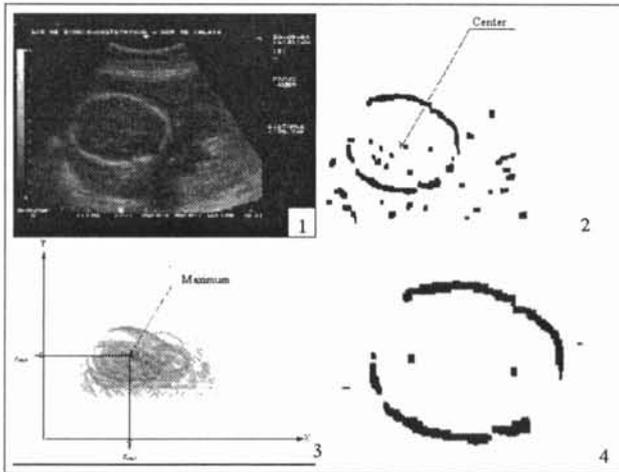


Figure 11: Foetal cranial outline (1), binary outline (2), image transformed by CSHT (3) and filtering image (4)

3.2 Image processing

We apply CSHT to the previous test image. It produces an image with the same dimensions as the test image. Gray levels represent the "vote". Each pair of points $\{(x_1, y_1), (x_2, y_2)\}$ correspond to a $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ dot in the transformed image. From the center detected, i.e. maximum on figure 11, we determine the points which led to this maximum. We obtain figure 11.

The transformed image is still noisy : our method use symmetrys to detect outline. So, if points which doesn't belong to the outline are symmetrical, they'll be keep. However, a modelisation method, like [2], allows to eliminate residual noise.

We can show evolution of center index on figure 12 : convergence speed increase with undersampled order and speed convergence is high even in the case of a one-order.

4 CONCLUSION

In this paper, we have showed that HT applied to the center research can be accelerate with two

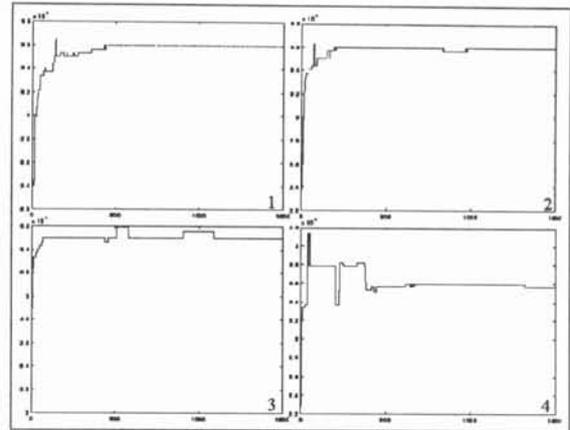


Figure 12: Evolution of center index for one, two, three and four-order undersampled

following methods : *image undersampled* and *convergence measurement*. In the case of foetal cranial contour, we have used this methods with the modelisation method described in [2]. Inside the scope of a parallel processing, let's note that maximum research is a independant operation.

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