# Train Wheel Abrasion Measurement Method using Contour Information

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# Abstract

We present a method to recognize a complicated curved object from an image contour. We are developing a system that automatically measures train wheel abrasion from its contour image. Because a train wheel consists of a curved surface, the transformation of the image contour due to position variation affects the measurement accuracy. The 3D shape model of the train wheel is analyzed to compute on image contour model represented with a numerical equation. The system corrects the measured value using the relation between position variation and contour model. Experimental result shows that the system can measure train wheel abrasion with an objective accuracy.

### 1 Introduction

We are developing a train maintenance system that automatically takes images of the wheels of a running train, and measures wheel abrasion from the deformation of wheel image contours.

Because a train wheel is worn down due to friction against the rail, some maintenance work to reshape the train wheel is needed when the abrasion becomes larger than a predetermined threshold value.

A method to measure wheel shape using sheet light has been proposed[1] previously. A system using sheet light has difficulty in determining the lighting conditions, because a wheel surface has specular reflection. Our method, using image contours, can easily determine the lighting conditions and has the following advantage compared to this previous method. The use of image taken under normal lighting conditions in our system allows an operator to observe the wheel surfaces directly, and hence other defects on wheels are detectable.

The problem in our system is that the measurement accuracy of the system is affected by the position variation of the train wheel. Because a train wheel consists of a smooth curved surface, the image contour changes due to both the position variation and abrasion. The positional error of a train wheel is corrected using the method proposed here. The wheel position is estimated from the image contour, and the abrasion value is corrected using the estimated wheel position.

## 2 Equation of wheel contour

Fig.1 shows a cross section of a train wheel. A train wheel is a solid of revolution and so consists of a cross section rotating around an axis. The right flat region of the flange in a Train wheel section makes contact with the rail and abrades due to friction. But, the left region of the flange in the wheel section does not abrade, as the region is not in contact with anything. Consequently, the wheel position is estimated from the image contour of the left region of the flange.



Figure 1: Cross section of a train wheel.

In our system, a 3D shape model of train wheel is represented by truncated cones piece-wise along the axis of rotation.

Ponce et.al. [2] proposed a position recognition method for parallel projected torus shapes, but, many objects in industrial applications have a complicated shape, and a piecewise object representation is more appropriate. Suppose point p = (x, y)on the image contour of a train wheel is the projection of a 3D point  $P = (x_3, y_3, z_3)$  on the wheel surface. These points p and P must satisfy all of the following conditions;

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1. 3D coordinate P is on the surface of a truncated cone represented by some parameters( such as center coordinate and shape of the truncated cone).

Assume the center of a truncated cone is  $(x_0, y_0, z_0)$ , and the rotation axis is parallel to the xaxis. A 3D coordinate P on the surface of the truncated cone satisfies the following equation.

$$((a(x_3 - x_0) + b)^2 = (y_3 - y_0)^2 + (z_3 - z_0)^2 (1)$$

where a and b are parameters representing the radius and the slope of the truncated cone, respectively

2. The normal vector to the surface of the truncated cone at P and the viewing vector from the camera to the coordinate at P are perpendicular.

The outer product of the tangent vectors on the surface result in the following normal vector to the surface.

$$(-ab + a^2(x_3 - x_0), y_3 - y_0, z_3 - z_0)^T \qquad (2)$$

Suppose the camera is at the origin. Then  $(x_3, y_3, z_3)^T$  is the viewing vector from the camera to the coordinate. Consequently, condition 2 can be represented by the following equation.

$$-a(x_3(b+a(-x_0+x_3))) + y_3(-y_0+y_3) + z_3(-z_0+z_3) = 0$$
(3)

3. p is the perspective projection of P onto the camera image plane.

Fig.2 shows the camera coordinate system. The camera is at the origin O, and the view direction is aligned with the z axis of the 3D coordinate. A 3D point P is projected onto point p in the image plane by a camera with focal length f. On this coordinate, p and P satisfy the following perspective projection equations.

$$\frac{x}{f} = \frac{x_3}{z_3} \tag{4}$$

$$\frac{y}{f} = \frac{y_3}{z_3} \tag{5}$$

The projection to the image of a truncated cone satisfies the simultaneous equations(1), (3), (4), (5). Therefore, the following equation(6) representing an image contour of a truncated cone is derived by simplifying and eliminating  $x_3, y_3, z_3$  from the previous equations using symbolic calculation software.

The solution to this equation gives two values of x for each y, but only one value will be in the field of view define by the image.



Figure 2: Camera coordinate system.

$$y = (-y_0(-2abx + 2a^2xx_0 - 2fz_0) \\ \pm SQRT(y_0^2(-2abx + a^2xx_0 - 2fz_0)^2 \\ -4(-b^2 + 2abx_0 - a^2x_0^2 + z_0)^2) \\ (-b^2f^2 + 2abf^2x_0 - a^2f^2x_0^2 + \\ f^2y_0^2 - a^2x^2y_0^2 - 2abfxz_0 + \\ 2a^2fxx_0z_0 - a^2x^2z_0^2)) \\ /2(-b^2 + 2abx_0 - a^2x_0^2 + z_0^2)$$
(6)

As mentioned before, the train wheel is represented by truncated cones piece-wise along the axis of rotation, and so the image contour model of the train wheel consists of piece-wise curve. Therefore, equation(6) shows the contour of each truncated cone for some ranges of x.

Because the rotation axis of the truncated cone is parallel to the X axis, the three dimensional surface of the truncated cone exists in certain range of X. Therefore, the following equation(7), representing the relation of the x coordinate of the image contour to the X coordinate on the surface of the truncated cone, can be derived by simplifying and eliminating  $y_3, z_3$  from the simultaneous equations (1),(3),(4).

$$\begin{aligned} \mathbf{x} &= (-2fx_3z_0((b-ax_0)(b-a(x_0-x_3))-y_0^2-z_0^2)\pm \\ &\quad 2\sqrt{f^2x_3^2(b-a(x_0-ax_3))^2y_0^2(-(b-ax_0)^2+y_0^2+z_0^2))}/\\ &\quad 2(b^4-4ab^3x_0+6a^2b^2x_0^2-4a^3bx_0^3+a^4x_0^4+2ab^3x_3-6a^2b^2x_0x_3+ \\ &\quad 6a^3bx_0^2x_3-2a^4x_0^3x_3+a^2b^2x_3^2-2a^3bx_0x_3^2+a^4x_0^2x_3^2-b^2y_0^2+ \\ &\quad 2abx_0y_0^2-a^2x_0^2y_0^2-2abx_3y_0^2+2a^2x_0x_3y_0^2-a^2x_3^2y_0^2- \\ &\quad 2b^2z_0^2+4abx_0z_0^2-2a^2x_0^2z_0^2-2abx_3z_0^2+2a^2x_0x_3z_0^2+ \\ &\quad y_0^2z_0^2+z_0^4) \end{aligned}$$

The derivative of the image contour described by the projection of piece-wise constant cones is not continuous at the contact point of the truncated cones. If the discontinuity is convex, there is a ridge between the truncated cones. The ridge between the truncated cones is represented with a circle, and the projection of circle onto the image is represented with the following equation.

$$\begin{aligned} x_0^2(f^2 + y^2) - 2xx_0(yy_0 + fz_0) &= \\ x^2(r^2 - y_0^2 - z_0^2) \end{aligned} \tag{8}$$

The image contour of the train wheel is represented with the contours of the truncated cones and ridges.

### 3 Measurement method

In manual measurement of abrasion, the train wheel abrasion is measured at a reference point shown with black circle in Fig.1 with a measuring tool. This system measures the train wheel abrasion directly from the image. The problem in this system is that the position of the reference point in the image changes by the position variation of the wheel. Because a train wheel consists of a smooth curved surface, the image contour changes due to the position variation.

Fig.3 shows a model of the image taken with the Maintenance system. The image is taken by the camera set under the rail. k shows the horizontal position of the crossing point of the wheel contour and pre-determined raster line, k + c shows the x position of the reference point in the image, and h shows the vertical distance between the top of the flange and the reference point. The difference of measured h value from the standard h value shows the abrasion of the train wheel. A problem that we solve is that the x position of the reference point changes by the position variation of the wheel. Because a train wheel consists of curved surface, the parameter c changes by the position variation of the wheel.

The position variation of the train wheel has three rotational components and three translational components. But the rotational components are neglectable. Also, the system can correct the two translational components along Y and Z axis of Fig.3 from an image taken from another direction. Consequently, the train wheel position variation along Xaxis of the image is the only uncertain parameter.

Because the values of the other parameters such as shape parameter a, b,focal length f and a position on the image contour (x, y) are known in advance, we can solve the image contour equation(6) for  $X_0$ . Then, the x position of the reference point c can be found using the  $X_0$  value and the equation(7).

Hence, the position of the reference point in the image is estimated and the train wheel abrasion can be measured at the estimated reference point.

This method can be easily applied to a different shaped wheel by changing the parameters to represent the new wheel shape.



Figure 3: parameters measured from the image.

## 4 Experimental Result

Fig.4 shows a computer generated train wheel image with the estimated contour superimposed on top of it. The generated contour accurately overlays the graphics contour at the left region of the flange. The cross section of the top region of the flange consists of a circle, and this region can't represented with truncated cones.

Fig.5 shows the relation between parameter k in Fig.3 and the X position variation of the wheel $X_0$  in this system. The relation of  $X_0$  and k can be approximated with a one dimensional linear equation  $k = 7.10x_0 + 229.42$ . This equation says that , the system correct the  $X_0$  with 0.15mm accuracy if the measurement error of k is less than 1 pixel.

Our method can easily generate such a graph from the analyzed equation.

Table 4 shows the accuracy of our method compared to the ordinary method over 18 experiments. In each experiment, the 3D wheel position varied from the standard position within the range of  $\pm 5mm$ .

The row entitled measurement tool shows the result using the ordinary measurement tool. In the before correction, the wheel is assumed on the standard position, and the system used fixed c value. In the after correction, the wheel position is estimated and the system used c value calculated by presented method.

The table shows the maximum value of the error from the value measured with this system and the mean value measured with the measurement tool and the standard deviation of the error.

The presented method can improve measurement accuracy, and the system can satisfy an objective accuracy of 0.5mm.

Table	1:	experimental	result
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	measurement error(mm)		
method	maximum	standard deviation	
before correction	0.42	0.20	
after correction	0.32	0.19	
measurement tool	-	0.15	



We have presented a method to recognize a complicated curved object from an image contour. The train wheel, which is the object in our system, is represented with truncated cones piece-wise along the axis of rotation. Equations that represent the projection of the truncated cone onto the image are analyzed to an equation that represent the image contour. The system estimates the train position from this equation, and corrects the measured abrasion value. Experimental result shows that the system can satisfy an objective accuracy.

## References

- H. Sasama, "Evaluation of Measuring Method by Sheet Light for Facilities of Railway", 8th Symposium on Sensing via Image Information, pp.57-62, '93
- [2] J, Ponce and D. Kriegman, "On Recognizing and Positioning Curved 3D OBjects from Image Contours", Image Understanding Workshop '89, pp.461–470.



Figure 4: superimposed contcur image on the graphics



Figure 5: relation of wheel position and k