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Matching for Affine Transformed Pictures using Hough Planes

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Abstract

Considering the relationship between the deformation on Hough planes and the affine transformation on raw images, the authors have proposed a matching algorithm ; four-dimensional optimization problem to search rotation, scale, and two-translation parameters has been decomposed into two-stages of one-dimensional and one stage of two-dimensional optimization problems. The algorithm achieves the reduction of calculating cost to search the four affine parameters. Furthermore, it works well on the images like human faces which are difficult to extract the feature points or segments since the Hough planes reflect global structure of straight lines on the raw images.

1. Introduction

Pattern matching is a fundamental problem in computer vision and it is applicable for many engineering purposes. In many cases, affine transformation is used to describe the correspondence between two images which are partially identical. There are many pattern matching methods, for example, template matchings, analytical methods using the optical flow, or generalized Hough transformation[2]. In these, the method presented by Kawakami detects translation by comparing two Hough planes [1]. It does not require a large voting space.

This paper presents a pattern matching method, into which Kawakami 's method is extended to estimate four affine parameters. That is, it can estimate the rotation and the scale change in addition to the translation. Section 2 discusses the formulation of a relationship between affine transformation and Hough transformation. Section 3 gives an algorithm to estimate four affine parameters using the formula derived in section 2. The crucial point is that a four-dimensional search problem is decomposed into one-dimensional and two-dimensional search problems by comparing two Hough planes. Therefore, it achieves a great reduction of calculation. Experimental results show that this method works well on the actual images.

2. Formulation of Relationship between Hough Transformation and Four Affine Parameters

The Hough transformation had been developed by Hough for detecting straight lines, and then, had been modified by Duda and Hart to a current style[3]. The straight line is represented by

$$\rho = x\cos\theta + y\sin\theta \qquad (1)$$

where θ is the angle of a line at inclination with respect to the x-axis, and ρ is the distance of the line from the origin. The Hough transform for detecting straight line requires a two-dimensional accumulator array $H(\rho,\theta)$ called Hough plane. Edge pixels, which have strong gradient, on the input image are voted to the Hough plane $H(\rho,\theta)$ by using equation (1). A set of collinear points on the original image ideally makes an integrated point on the Hough plane.

Stanley has shown that the Hough transform is a kind of the Radon transform[4]. Let $I(x,y)$ be an edge image defined on the x-y plane. The Hough transform

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is given by

$$H(\rho,\theta)=\iint I(x,y)\delta[\rho-x\cos\theta-y\sin\theta]dxdy \qquad (2)$$

where $\delta(\bullet)$ is Direc delta function. Consider two images I_1 and I_2 which are related each other by four affine parameters as follow:

$$I_1(x,y)=I_2(x',y') \qquad (3)$$

$$\begin{pmatrix}x'\\y'\end{pmatrix}=s\begin{pmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}t_x\\t_y\end{pmatrix} \qquad (4)$$

where s is the scale parameter, φ is the rotation parameter, t_x and t_y are the translation parameters. Based on equation (2), two Hough planes $H_1(x,y)$ and $H_2(x,y)$ are written as

$$H_1(\rho,\theta)=\iint_D I_1(x,y)\delta[\rho-x\cos\theta-y\sin\theta]dxdy \qquad (5)$$

$$H_2(\rho,\theta)=\iint_D I_2(x',y')\delta[\rho-x'\cos\theta-y'\sin\theta]dx'dy' \qquad (6)$$

By substituting equation (3) and (4) into equation (5), these two Hough planes are related by the four affine parameters.

$$H_2(\rho,\theta)=H_1\left(\frac{\rho-t_x\cos\theta-t_y\sin\theta}{s},\theta-\varphi\right) \qquad (7)$$

It should be noted that the rotation is expressed as a shift along the θ-axis, and the scale change and the translation appear as a scale change and a shift along the ρ-axis for each θ.

3. Estimation of Four Affine Parameters

The authors presents an algorithm to estimate the four affine transform parameters from equation (7). The four affine transform parameters are estimated sequentially in the order of (1)rotation, (2)scale, (3)translation. If necessary, for more accurate estimation, the estimation can be iterated recursively using the image estimated last time. The sequence of this method is shown in Figure 1.

3.1 Estimation of Rotation Parameter

Projecting the Hough planes to the θ-axis gives directional histograms written as

$$D_i(\theta)=\int H_i(\rho,\theta)d\rho \ , \ i=1,2 \qquad (8)$$

Considering the fact that the rotation parameter φ corresponds to the shift along the θ-axis from equation (7), two directional histograms are related by

$$D_2(\theta)=D_1(\theta-\varphi) \qquad (9)$$

Therefore, the rotation parameter $\hat{\varphi}$ is estimated by evaluating the residual between the two directional histograms D_1 and D_2. Next, by using the estimated

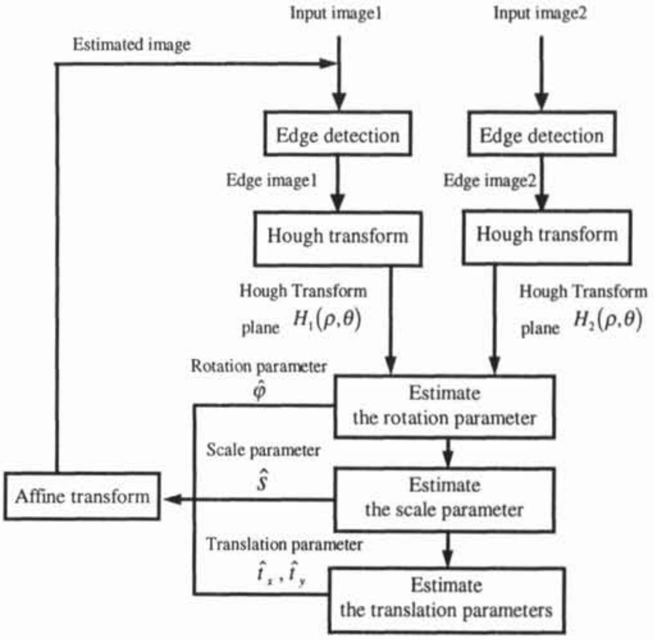

Figure 1 The flowchart

rotation parameter $\hat{\varphi}$, the Hough plane $H_1(x,y)$ is reformed by shifting along the θ-axis. That is,

$$H_2(\rho,\theta)=H_1'\left(\frac{\rho-t_x\cos\theta-t_y\sin\theta}{s},\theta\right) \qquad (10)$$

3.2. Estimation of Scale Parameter

From equation (10), we find that the profiles of the two Hough planes along ρ-axis for each θ have similar shapes except for the scale change and translation. The scale parameter between the two profiles is equal to that of input image. Therefore, the scale parameter is estimated by finding the scale between these two profiles. Then, the scale parameter $\hat{s}$ for the whole image is ,e.g., given by averaging the estimated scale parameters for all θs.

3.3. Estimation of Translation Parameters

The translation t_x and t_y between the two images is expressed as the shift t_a of the profile of the Hough plane along ρ-axis for each θ. The shift t_a is obtained by comparing the two profiles as in the section 3.2. Here, it should be noted that the shift t_a is the observed value, and is related to the actual translation t_x and t_y by

$$t_a = t_x \cos\theta - t_y \sin\theta \qquad (11)$$

Hence, equation (11) serves as a regression equation, and the two parameter, $\hat{t}_x$ and $\hat{t}_y$ are estimated by applying a least squares method to the data set $\{t_a(\theta_1),\theta_1\},\{t_a(\theta_2),\theta_2\},\cdots$.

4. Experimental Results

Figure 2 shows an experimental result for the proposed matching algorithm. These two input images are related by affine parameters $(s = 0.75, \theta = 60, t_x = 10, t_y = 20)$. Figure 2(c) is the estimated image created from the input image1 using the estimated parameters $(\hat{s} = 0.75, \hat{\varphi} = 61, \hat{t}_x = 9, \hat{t}_y = 21)$, and Figure 2(d) is residual image between estimated image and input image 2. The residual image shows that the estimation succeeded in one pixel accuracy. It takes one second to execute the whole processes using SUN SparcStation 10, which includes edge extraction and Hough transformation. Figure 3 shows the relation between the image size and the calculating time. The calculating cost of the estimation increases slowly comparing with that of the edge extraction. As a result, the total cost is proportional to the size of the input image.

The following experiment shows how large difference between two input images this method can be applied to. The two input images are cut from one original image(See Figure 4). As a criterion of the difference between two input images, the rate of overlapping is defined as follow:

$$p = \frac{S_{12}}{S} \quad S = \max\{S_1, S_2\} \qquad (12)$$

where S is the area of the larger image, and S_{12} is the area which is shared by the two input images S_1 and S_2. The pictures of human face and sight are used as an original image. Figure 5 shows the result. The vertical axis is the residual of intensity per pixel between the estimated image and the input image, and each point indicates a result of matching. In the case that the residual is over 40, estimated affine parameters are apart from true values. From Figure 5, we find that if the rate of overlapping is lager than 0.7, then the matching succeeds.

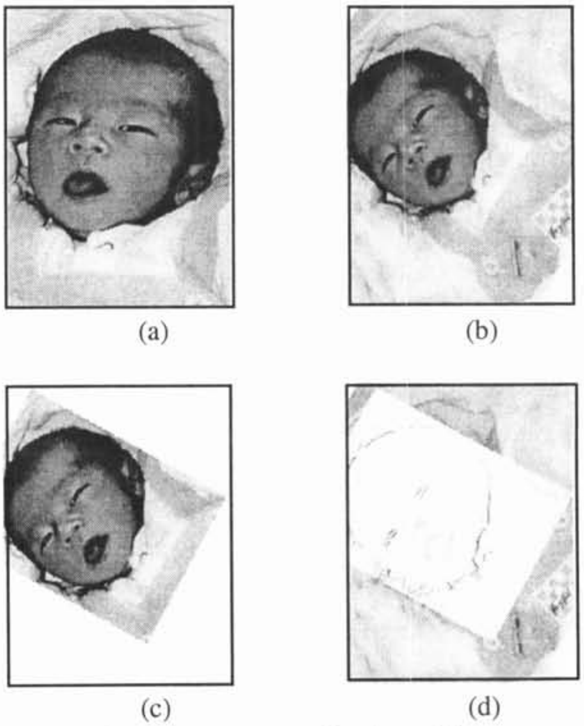

Figure2 An example of matching
(a) input image1(150x200), (b) input image 2, (c)estimated image, (d) residual image between estimated image and input image 2.

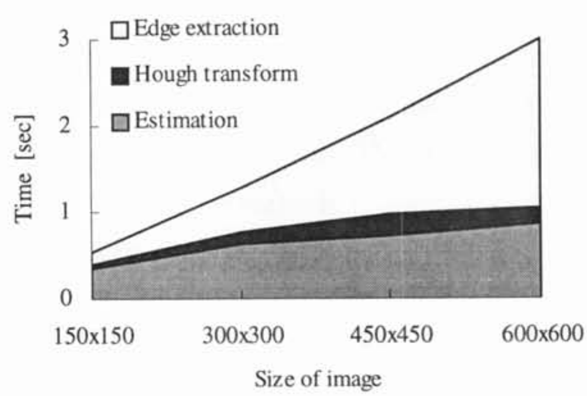

Figure 3 Time for estimation

5. Conclusion

A pattern matching method using Hough planes was presented. Hence, the relationship between the affine transform and the Hough transform can be written as a simple formula, it became easier to estimate the affine parameters on the Hough planes than on the original images. This method is much faster than four dimensional searching methods, and were successful to match the image like a human face from which it is difficult to extract feature points. For the future work, we will extend this method to 3-dimentional object matching.

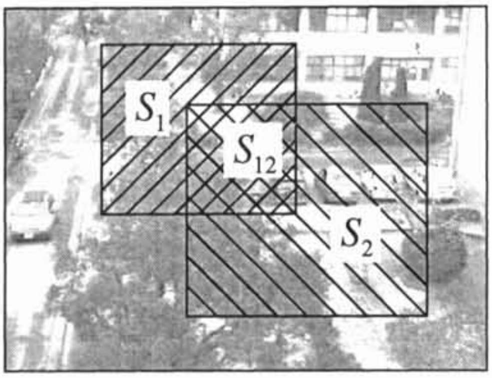

Figure 4 Two input images cut from one image.

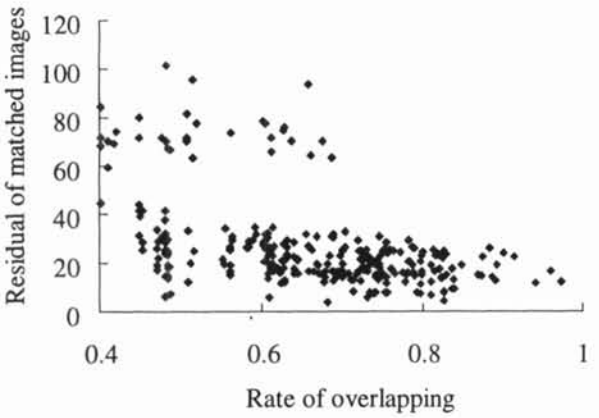

Figure 5 Result of matching experiments

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