

Shape Recovery for Input of a Rough 3D Shape by its 2D Sketch<sup>‡</sup>

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### Abstract

A sketch is often used as a simple and useful medium to communicate a rough 3D shape among humans. In order to realize man-machine communication of rough 3D shape using a sketch, we have to discuss the problem how to recover the 3D shape implied by a human using a 2D sketch. In this paper, we propose to recover the 3D shape from its sketch interactively based on some qualitative features that would be important to characterize 3D shapes implied with sketches. We also propose to infer the 3D shape that is not drawn in the sketch from the shape recovered from the drawn part.

### 1 Introduction

A monocular sketch is often used as a simple and familiar medium to communicate a rough 3D shape among humans in our daily life. In order to realize such communication between a human and a computer, we have to solve the problem of *shape recovery from a monocular image*, which is one of the conventional problems in computer vision. This problem has long been discussed as one of the key issues for recognizing an object in the 3D world from its 2D image taken by a camera[1][2]. In this paper, we discuss the problem from a new viewpoint, aiming to realize a method of man-machine communication of rough 3D shapes by its 2D sketch.

One of the points to be considered in a discussion on shape recovery is what to be recovered from the image depends on the task. In the previous work on shape recovery, it has been aimed to recover the original quantitative precise 3D shape of an object in real world, in order to recognize the object. On the other hand, assuming to recover a 3D shape for

man-machine communication, what to be recovered from a 2D sketch is the 3D shape implied by a human with the sketch. This 3D shape is a *mental image* that is not detailed enough to be described as a fully quantitative 3D shape. It makes no sense to recover too much detailed shape, compared to mental image.

Another point to consider about shape recovery for man-machine communication is that it is insufficient only to recover the 3D shape that is drawn in a sketch. The previous work on shape recovery discusses how to recover the 3D shape of a part of an object whose 2D appearance is given as an input image, yet gives no consideration to recover the shape of the other part that does not appear in the image. But, when we communicate a 3D shape by a sketch as mentioned above, we often omit to draw the part that can be easily inferred from the drawn part in the sketch. Thus, in order to correctly understand the 3D shape that is implied by a human, it is required not only to recover the 3D shape of the drawn part, but also to infer the 3D shape that is not drawn in the sketch.

Considering these issues, we first propose a method to recover *qualitative 3D shapes* from a 2D sketch based on some features that characterize qualitative difference in shape. In more detail, 3D shapes with a few sorts of angles qualitatively different, as the right angle, acute angle, or obtuse angle, are recovered from a sketch. These recovered shapes are presented to the user as the candidates so the user can select the correct one from them. Second, we propose a method to infer the shape of the omitted part from that of the drawn parts which are assumed to be recovered by the method described by the method above. This inference is realized by a rule-based system with rules describing geometric and topological constraints to be satisfied in a proper 3D shape. We can obtain two or more probable shapes by *backtracking*. In section 2, 3, we describe these two methods in detail. In section 4, we present experimental results of these methods. Finally, in section 5, we discuss our future steps.

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‡ This research was funded in part by Grant-in-Aid for Scientific Research of the Ministry of Education, Science and Culture of Japan.

## 2 Shape Recovery Based on Qualitative Difference of Angles

### 2.1 Representing Recovered Shape by Gradient Space

We assume that 3D shapes to be recovered are polyhedra and their sketch are drawn in the orthographic projection. The 3D shapes that are proper as recovered shapes for a polygon of a sketch vary with their orientations, and the 3D shape can uniquely be recovered if its orientation is specified. Let us assume a coordinate system in which the origin is at the center of the image plane, and the  $x$ ,  $y$ ,  $z$ -axes coincide with the horizontal direction and the vertical direction of the image plane, and the optical axis, respectively. We call this coordinate system an *image-centered coordinate system*. (see Fig. 1).

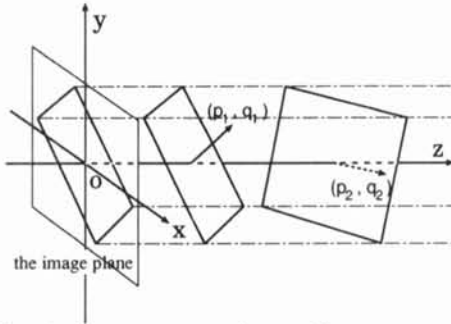


Fig. 1: image-centered coordinate system

We represent the orientation of a plane  $z = px + qy + K$  in this coordinate system by its parameters  $(p, q)$ . The parameters span a space called *gradient space*. Different recovered shapes of a 2D polygon in a sketch may correspond to different points in this gradient space.

### 2.2 Recovery of Qualitative Shapes

Let us assume two edges  $e_1, e_2$  that share a vertex of a polygon  $P$ . Let  $\alpha$  be the angle between these two edges, and let  $\beta, \gamma$ , be the angles between the  $x$ -axis and each of the two edges, respectively. Let  $\theta$  be the angle between  $e_1$  and  $e_2$  in a recovered shape (see Fig. 2).

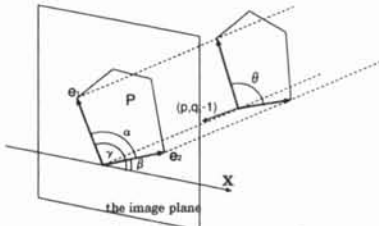


Fig. 2: An example for recovery about one angle.

The orientations  $(p, q)$  of a 3D shape recovered from  $P$  are represented as follows[3]:

$$\begin{aligned} (p' \cos \frac{\alpha}{2})^2 - (q' \sin \frac{\alpha}{2})^2 &= -\cos \alpha \\ &+ \cos \theta \sqrt{1 + (p' \cos \frac{\alpha}{2} - q' \sin \frac{\alpha}{2})^2} \\ &\cdot \sqrt{1 + (p' \cos \frac{\alpha}{2} + q' \sin \frac{\alpha}{2})^2} \end{aligned} \quad (1)$$

$$\text{where } \begin{pmatrix} p' \\ q' \end{pmatrix} = \begin{pmatrix} \cos \frac{\beta+\gamma}{2} & \sin \frac{\beta+\gamma}{2} \\ -\sin \frac{\beta+\gamma}{2} & \cos \frac{\beta+\gamma}{2} \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}$$

When  $\theta = \frac{\pi}{2}$ , these orientations are on the hyperbola shown in Fig. 3. When  $\theta > \frac{\pi}{2}$  or  $\theta < \frac{\pi}{2}$ , the orientation of a recovered shape is the inside and outside of the hyperbola.

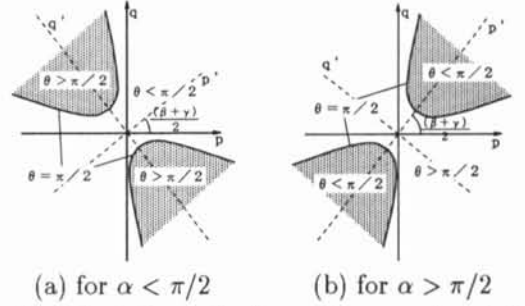


Fig. 3: Orientations of a face with an acute, right or obtuse angle.

When there is a polygon with  $n$  vertices in the sketch, we obtain  $n$  pairs of hyperbola corresponding to the angles of the vertices (see Fig. 4).

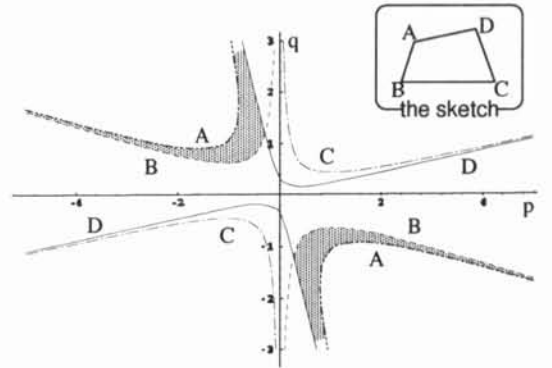


Fig. 4: The hyperbolas for a polygon.

An intersection of  $m$  different hyperbolas means that  $m$  angles become right angles at the same time. Each region bounded by the hyperbolas corresponds to the orientations in which the polygon has the same recovered shape with respect to qualitative difference in their angles: acute, obtuse or right. For example, the shaded regions of Fig. 4 meet the orientations of recovered shapes with the acute angle for vertex C and three obtuse angles for the others.

By selecting a representative point from each intersection of hyperbolas and each hyperbolic line segment or regions bounded by hyperbolas, we can

get qualitative 3D shapes to be regarded as candidates of the shape implied in the sketch. These candidates are presented to the user so that the correct one is selected interactively.

### 3 Inference of the Omitted Part

#### 3.1 Topological Constraints

The constitutional structure of faces, edges and vertices of the drawn part are given as that of regions, segments and vertices in the sketch, whereas nothing is given for the part omitted to draw. Thus, in order to infer from the shape of the drawn part recovered by the method in the previous section, we first need topological constraints to narrow down structures of its faces, edges and vertices. One of the constraints that would be useful for this inference is that to require the faces to constitute a polyhedral surface together with the drawn part. We define this constraint as follows:

Constraint 1:(Constraint to be a polyhedron)  
Each edge is shared by two faces. (Fig 5(a))

Constraint 2:(Constraints to be a polygon)  
1. Each vertices is shared by two edges on each face. (Fig 5(b))  
2. Each edge has two vertices at its endpoints. (Fig 5(c))

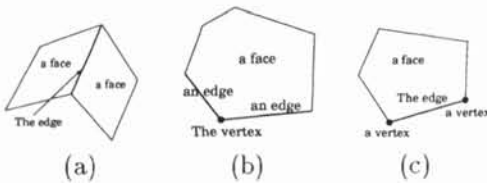


Fig. 5: Constraints to constitute a polyhedron

Assuming neither objects with a hole on their faces nor those with faces that intersect with each other more than once at different positions, we employ the following constraints.

Constraint 3:(Uniqueness of intersection between two faces)  
Arbitrary two faces share at most an edge.

Since these constraints are still insufficient to determine the topological structure of the omitted part, we introduce the following heuristic constraints.

Constraint 4:(Assumption to omitted faces)  
At least an edge of each face appears in the sketch.

Constraint 5:(trihedral-vertex constraint)  
The vertex is an intersection of three faces and three edges.

#### 3.2 Geometric Constraints

All the surface that satisfies above topological constraints do not have geometric shapes to be a proper polyhedron. To exclude the surface which is impossible to exist, we employ the following geometric constraints.

Constraint 6:  
The normal of a face and each edge on the face are orthogonal to each other.

Constraint 7:  

- The normals of two faces which shares an edge are not parallel to each other.
- Two edges by which a vertex is shared belong to the same face, and their directions are not parallel to each other.

#### 3.3 Rules for Inference

Inference on the shape of the omitted part based on the above topological and geometric constraints is executed by a rule-based system. The rules for the inference are described as follows.

- Rule 1 (Addition of faces based on constraint 1,4)  
Add a new face to each edge that belongs to only one face(Fig. 6(a)).
- Rule 2 (Merger of faces based on constraint 5)  
Merge each pair of newly added faces sharing a vertex that belongs to other two different faces(Fig. 6(b)).
- Rule 3 (Addition of edges based on constraint 5)  
Add an edge to each vertex that is shared less than three edges(Fig. 6(c)).
- Rule 4 (Merger of edges based on constraint 3)  
Merge each pair of edges shared by the same pair of faces(Fig. 6(d)).
- Rule 5 (Addition of vertices based on constraint 2)  
Add a new vertex to be shared by each pair of edges that belong to the same face and share no vertex(Fig. 6(d)).

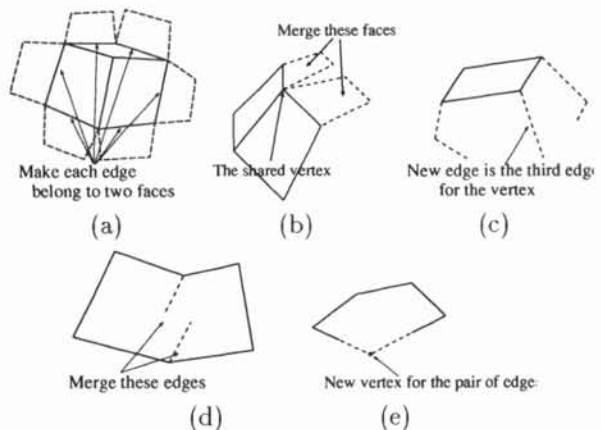


Fig. 6: Rules for inferring the omitted part.

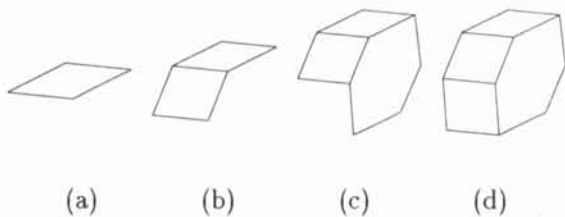


Fig. 7: Input sketches.

If a topological structure that has no geometric shape satisfying constraint 6 and 7, is generated by one of the rule above, firing the rule is rejected and another rule is tried by backtracking. The rule described earlier above has higher priority to be fired.

#### 4 Experimental Results

Fig. 7 and 8 show the input sketch by a user and some examples of shapes recovered by the method described in section 2 as the candidates for the shape implied by the user. Fig. 8(a)-(c) are examples recovered from Fig. 7(a). Fig. 8(d)-(f) are those recovered from Fig. 7(b), assuming that the user selected the shape in (a) as the proper one. Similarly, Fig. 8(g),(h) are shapes recovered from Fig. 7(c),(d) after (d) is selected from among (d)-(f). Note that the possible shape is limited further, when shapes are recovered for a polygon after the 3D shape of its adjacent polygon is recovered.

Fig. 9 shows the result of inferring the 3D shape of the omitted part of Fig. 8(h) by the method described in section 3. Fig. 9(a),(b) topologically correspond to a hexagonal prism and a pentagonal prism with a vertex cut off, respectively. Note that there sometimes exist parts whose geometric shapes not completely determined because of the lack of geometric constraints under these rules. In Fig. 9, the dotted lines denote edges whose geometric information cannot be determined uniquely. In such a case, the system require the user to give more drawings.

#### 5 Conclusion

We proposed a method of shape recovery from a sequence of sketches, as rough figures of faces, with angles in a qualitative expression by means of man-machine interaction and inference of structures of occluded faces.

Different from conventional shape recovery for object recognition, this work aimed to realize man-machine communication of 3D shape by a 2D sketch like the communication among humans. The sketches can only carry the information about not the details of a shape but the features of it, because they merely represent a surface with rough angles in qualitative expression. To cope with this input vagueness we proposed an interactive method of shape recovery by means of showing possible shapes generated by combination of plausible faces with orientations calculated from the sketches and testing by

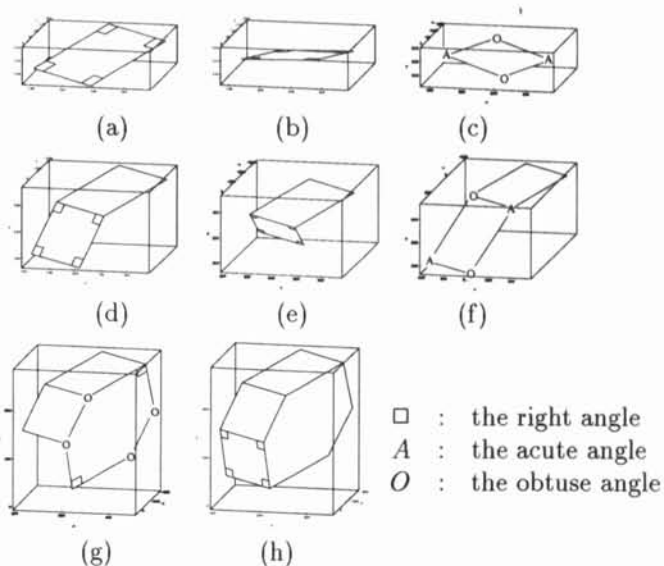


Fig. 8: Recovered 3D shapes.

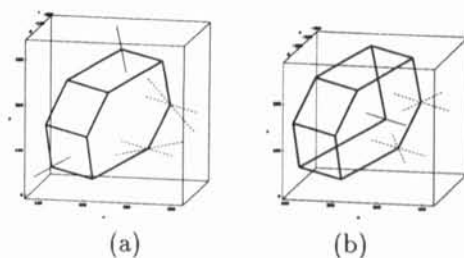


Fig. 9: Inference the 3D shape of the omitted part.

the user's choice of one of them. We also attached a mechanism of inferring the parts omitted in the input from the selected surface in the interaction.

In order to recover more characterized shapes from the sketch, we are introducing some other kinds of features for input items such as parallelism of edges or faces. Also the given information about the visible parts should be used to supplement the lacked information for inferring the shape of omitted parts. Finally, in order to construct the whole system applying these methods, it is necessary to consider the GUI protocol. For this problem, we will discuss how to modify the error in drawing the sketch, how to display the results for ease to grasp a shape and so forth.

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