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IMAGE SEGMENTATION BASED ON FUZZY CLUSTERING ALGORITHM

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ABSTRACT

Image segmentation plays an important role for machine vision applications. In this paper, we present a new segmentation strategy based on fuzzy clustering algorithm. The new algorithm includes the spatial interactions by assuming that the statistical model of segmented image regions is Gibbs Random Field (GRF). We specify the neighborhood system, the associated cliques, and the potentials of the GRF. Then, we redefine the objective function of Fuzzy C-Means (FCM) clustering algorithm to include the energy function that is the sum of potentials. The modified membership equation is derived. By including the modified membership equation in the modified FCM clustering algorithm, the segmentation is achieved. Experiment results show that the new algorithm yields better segmentation results. Moreover, it is faster than the conventional adaptive segmentation algorithm.

I. INTRODUCTION

Image segmentation plays an important role for machine vision application. It aims to partition an image into a set of nonoverlapping regions whose union is the original image. The regions that appear the same would produce the corresponding features that are near to each other, whereas regions that appear different would produce the corresponding features that are far apart [1]. Consequently, the process of segmenting an image is equivalent to the process of grouping image samples with similar features into regions (clusters). Thus, it can be considered as a clustering problem.

Clustering is the process of partitioning a set of feature vectors into clusters [2]. There are many clustering algorithms, each having its own peculiar characteristics. However, they can be roughly categorized into two board types : 1) "hard" clustering algorithm and 2) "soft" (or "fuzzy") clustering algorithm. Most of the early works in image segmentation by clustering are based on the hard clustering algorithm. A well-known hard clustering algorithm is the K- Means algorithm [3] which will iterate to a local minimum for the squared errors (distances), from each sample to the nearest cluster center. However, detailed studies of the K-means have revealed the following two major problems :

1) it assigns each pixel to one and only one cluster during each iteration. However, the "all or none" membership restriction is not realistic one.

2) it doesn't include spatial constriants which is important to remove the isolated pixels.

In order to solve the second problem above, an algorithm (hereafter called the adaptive algorithm) was recently proposed by Pappas [4]. This adaptive algorithm can be regarded as a generalization of K-means algorithm. It aims to contain the spatial constraints at the cost of expensive computational burden.

The fuzzy clustering method assigns each training vector a set of membership values, one for each cluster, rather than assigning each training vector to one and only one cluster. Since it is more realistic, several research results have indicated that it is superior to the hard clustering algorithm.

The most popular algorithm in the fuzzy clustering is the Fuzzy C-Means (FCM) algorithm [5]. Our study indicated that FCM has a major problem : a large amount of storage requirement. In order to overcome this problem, we have developed a modified version of FCM (hereafter called MFCM) which uses a recursive procedure, rather than a batch procedure used in FCM, to update cluster centers [6]. We applied the MFCM to the image compression problem and demonstrated that it can reduce the storage significantly.

In this work, we develop an image segmentation algorithm based on the MFCM clustering algorithm. The MFCM (or FCM) clusters each image pixel (sample) without the use of spatial constraints. To improve the segmentation, we modify the objective function of the FCM to include spatial constraints. We make use of the spatial constraints by assuming that the statistical model of image clusters is the Markov Random Field (MRF). The MRF has aroused wide attention in recent years, which has proven to be a powerful modeling tool in several aspects of image processing. This is due to the ability of MRFs to model image joint distribution in terms of local spatial interactions, through their formulation as Gibbs distribution (GD). We employ the energy function of the GD to modify the objective function of FCM. Then the modified membership equation is derived. Finally, the segmentation is achieved by including the modified membership equation in the MFCM clustering algorithm which uses an iterative procedure to find the optimal membership values and cluster centers. The proposed strategy then assigns each pixel to a cluster having the largest membership value.

We compare the proposed strategy with K-means and the adaptive algorithm. Experiment results indicates that the performance is improved significantly. Moreover, it is faster than the adaptive algorithm.

II.IMAGE MODEL

In this section, we present a description of the assumed image model. All images are defined on a M× N rectangular lattice L={ (r,s): $1 \le r \le M, 1 \le s \le N$ }. The observed gray scale image is $y=\{y_{rs}\}$. A segmentation of the image y into region is denoted by $X_{rs} = i$, where X_{rs} take values in Q={ 1, 2, - - -, M }. M is the different region types (or clusters). $X_{rs}= i$ denotes that the pixel at (r,s) belongs to region i. For simplicity, X_k^i is used to denote that the pixel at location k belongs to region i. Thus a segmentation is simply a partition of the image y into M region types. Each region type can occur in more than one location within the image.

We assume that the region process is a MRF with respect to a neighborhood system $\eta=\{\eta_k : k \in L\}$, and then the conditional density is described in terms of the local characteristics

$$\begin{aligned} &P(X_k | X_q, \text{ all } k \neq q) \\ &= P(X_k | X_q, q \in \eta_k) \end{aligned} \tag{1}$$

where η_k is the neighborhood of pixel at location k. The neighborhood systems that are commonly used in image processing are η^1 and η^2. η^1 denotes the first-order neighborhood system consisting of the four nearest pixels. η^2 means the second-order neighborhood system consisting of the eight nearest pixels. In this paper, the second order neighborhood system is adopted. Since a one-to-one correspondence exists between MRFs and GRFs [7], each region process can be respresented by its Gibbs Distribution (GD)

$$P(X_k^i) = \frac{1}{z}\exp[-E(X_k^i)] \tag{2}$$

where

$$E(X_k^i) = \sum_{X_k \in B} V_B(X_k^i) \tag{3}$$

B is a clique.

Z is simply a normalizing constant.

$V_B(X_k^i)$ = potential associated with clique B.

$E(X_k^i)$ is called the energy function. A clique is a set of pixels that are neighbors of each other. The clique types associated with the first order and second order neighborhood system are shown in Fig.1 respectively. In this paper, we assume that the only nonzero potentials are those associated with single pixel and two pixels cliques. The potential for two pixels clique is defined as

$$V_B(X_k^i) = \begin{cases} 0, & \text{if } X_k = X_q = i \text{ and } k, q \in B \\ +\beta, & \text{otherwise} \end{cases} \tag{4}$$

where the parameter β influences the size and shapes of the resulting regions. For the single clique, the potential is defined as

$$V_B(X_K^i) = \alpha_i \text{ if } X_k = i \text{ for } k \in B \tag{5}$$

where α_i is a parameter associated with region i. The parameter α_i influences the relative likelihood of each region type. In this paper, we assume that all region types are equally likely. In other word, the parameter α_i equals to zero for all i.

One would expect to obtain better segmentation results by considering higher order neighborhood system and all associated clique types. However, this will increase the computational complexity significiantly. Experiment results indicated that our model captures the essential features of the region in an image and yields good segmented images.

III. OBJECTIVE FUNCTION CLUSTERING

In this section, we first review the fuzzy objective function and point out its drawback. Then we describe how to modify the fuzzy objective function to solve this drawback.

A. The Fuzzy Objective Function

Let $y=\{y_1, y_2, \ldots, y_n\}$ be the observed grey scale image , and an integer C, $2 \le C \le n$, be the number of clusters (or regions). The fuzzy clustering algorithm attempts to partition y into C fuzzy clusters, such that close elements in y will have similar segmentation (membership value) and dissimilar elements will have different segmentation. The clustering of a pixel y_k depends on the membership vector $\boldsymbol{u}$ = $[\mu_1(y_k), \mu_2(y_k), \ldots, \mu_C(y_k)]^T$, where $\mu_i(y_k)$ (denoted as μ_{ik} for simplicity), $1 \le i \le C$, indicate the degree of belonging of y_k in the fuzzy region i. In order to achieve optimal clustering, we can define an objective function as

$$J_m(U, V) = \sum_{k=1}^{n}\sum_{i=1}^{C} (\mu_{ik})^m (d_{ik})^2 \tag{6}$$

The parameter $m \in [1, \infty)$ is the weighting exponent; U= $[\mu_{ik}]$ is the membership matrix with dimension $C\times$ n; V = $\{v_1, v_2, \ldots, v_c\}$ is a set of cluster centers; $d_{ik} = \|y_k - v_i\|$, where $\|\bullet\|$ is any inner product norm metric, denotes the distance between y_k and center v_i. Ruspini [8] interprets the objective function, J_m(U,V), as a clustering criterion and optimal fuzzy C-partitionings of y are taken as local minima of J_m(U ,V). Since the membership values, μ_{ik}, are restricted by the following conditions :

1) $\mu_{ik} \in [0, 1]$
2) $\sum_{i=1}^{C} \mu_{ik} = 1, \forall k$

One can uses Lagrange's method to solve this contrained optimization, and then obtains

$$\mu_{ik} = \frac{1}{\sum_{j=1}^{C}\left(\frac{d_{ik}}{d_{jk}}\right)^{\frac{2}{m-1}}}, \ \forall i, k \tag{7}$$

$$v_i = \frac{\sum_{k=1}^{n} \mu_{ik}^m y_k}{\sum_{k=1}^{n} \mu_{ik}^m}, \ \forall i \tag{8}$$

These main results are contained in the FCM clustering algorithm which uses an iterative procedure to find the optimal sets of U and V. However, it is clearly that the objective function doesn't include spatial constraint component. This is the same drawback that the K-means algorithm has. In order to overcome this drawback, we attempt to modify the fuzzy objective function to contain the spatial constraints.

B. The Modified Fuzzy Objective Function

In this work, we will use the energy function to introduce the spatial constraints. The energy function $E(X_k^i)$ expressed in Eq.(3) is the sum of the local potential terms which consist of a linear combination of the self-information of the current pixel and the mutual information between this pixel and its neighbors. The smaller $E(X_k^i)$ is the more likely that the pixel X_k belongs to region i. Thus the spatial constraints of each pixel can be introduced through its associated energy function.

By considering the energy function as a weighting component, we can define a weighting function as

$$W_{ik} = (\mu_{ik})^m E(X_k^i) = (\mu_{ik})^m \sum_{X_k \in B} V_B(X_k^i) \quad (9)$$

Referring to fuzzy objective function (Eq.(6)) and containing the weighting function term in Eq.(9), we obtain

$$J_m(U,V) = \sum_{k=1}^{n}\sum_{i=1}^{C} (\mu_{ik})^m[(d_{ik})^2 + W_{ik}] = \sum_{k=1}^{n}\sum_{i=1}^{C} (\mu_{ik})^m[(d_{ik})^2 + \sum_{X_k \in B} V_B(X_k^i)] \quad (10)$$

This modified objective function simply adds the weighting component to the square distance between each pixel and a region center; the squared distance is then weighted by the spatial constraints.

Using the same solving technique described in Section III(A), we obtain the modified membership equation

$$\mu_{ik} = \frac{1}{\sum_{j=1}^{C}\left[\frac{(d_{ik})^2 + \sum_{X_k \in B} V_B(X_k^i)}{(d_{jk})^2 + \sum_{X_k \in B} V_B(X_k^j)}\right]^{\frac{1}{m-1}}}, \forall\, i,k \quad (11)$$

This main result will be included in the following modified FCM clustering algorithm, and then the segmentation is achieved.

IV. ALGORITHM

In this section, we use a modified version of FCM (hereafter called the MFCM) to reduce the storage requirement. The main idea behind the MFCM is that it uses a recursive procedure, rather than a batch procedure used in FCM, to update the cluster center. The segmentation is achieved by including the modified membership equation in the MFCM algorithm which uses an iterative procedure to find the optimal membership values and cluster centers. Then, it assigns each pixel to a cluster having the largest membership value. The algorithm is described as follow.

The Algorithm

(A1) Given an observed grey scale image $y=\{y_1, y_2, \cdots, y_n\}$ where y_k is a pixel at location k. Fix C, $C \in [2,n)$; fix m, m$\in [1,\infty)$.

Where

C = the number of clusters .

n = the number of pixels

m = the weighting exponent.

(A2) Fix the order of neighborhood system and the associated clique potentials.

(A3) Initialize the cluster centers $V^{(0)}=\{v_i^{(0)}; 1 \le i \le C\}$ .Let l=0, k=1; clear a_i and b_i for $1 \le i \le C$.

(A4) For each i, calculate the membership values μ_{ik} using the modified membership equation (Eq.(11)). The membership values are then stored into a temporary linear array $\mu = [\mu_1, \mu_2, \cdots, \mu_C]$.

(A5) For each i, calculate two intermediate parameters, a_i and b_i by

$$a_i^{(l)} = a_i^{(l)} + \mu_i^m X_k \quad (12)$$

$$b_i^{(l)} = b_i^{(l)} + \mu_i^m \quad (13)$$

(A6) Set $k = k+1$ and return to (A4) until all pixels (k = n) have been processed. Then, go to (A7).

(A7) For each i, update the cluster centers using $a_i^{(l)}$ and $b_i^{(l)}$

$$v_i^{(l+1)} = \frac{a_i^{(l)}}{b_i^{(l)}} \quad (14)$$

(A8) Compare $v^{(l+1)}$ to $v^{(l)}$ in 2-norm. If $\|v^{(l+1)} - v^{(l)}\| \le \varepsilon$ the procedure stops and goes to (A9). Otherwise, set $a_i = 0$, $b_i = 0$, l= l+1, and go to (A4) for next iteration.

(A9) Assign each pixel to a cluster having the largest membership value.

In MFCM, only the membership values corresponding to one pixel are needed to be stored. Hence, the gain in memory saving is about equal to n, where n is the number of all pixels. Furthermore, the MFCM achieves a better segmentation performance. The performance advantage will be demonstrated in the experiment.

V. EXPERIMENT RESULTS

The performance of the proposed algorithm is evaluated on a PC486-50. Several natural images with 256 grey levels are used. The segmentation is set to be 4 clusters. The second order neighborhood system is adopted. The weighting exponent m is set to 1.5. We compare the proposed algorithm with the K-means algorithm, and the adaptive algorithm. The results of segmented image are presented in Fig.2. Fig.2(b) shows the results of the K-means. A lot of isolated pixels on the wall and shadow regions (e.g., upper right corner of the door) are lost. Fig.2(c) shows that the adaptive algorithm gives less isolated pixels. However, much of the shadow regions are still lost. The result of the proposed algorithm, shown in Fig.2(d), is obviously better than K-means and the adaptive algorithm.

Table 1 compares the computation times of K-means, the adaptive algorithm, and the proposed algorithm. The results indicated that K-means is the fastest one. However. the quality of segmented image processed by K-means is poor. It is reasonable because K-means uses no spatial constraints. The proposed algorithm is obviously faster than the adaptive algorithm.

VI. CONCLUSIONS

In this paper, we have presented a new strategy to segment an image, based on the MFCM clustering algorithm. In this strategy, a MRF is used to model the image. Then, the associated cliques and the associated potentials are used to form the energy function which is used to modify the fuzzy objective function. Hence, the modified objective function has two components. One measures the dissimilarity between pixels and cluster centers, and the other introduces the spatial constraints. Finally, a modified membership equation is included in the MFCM clustering algorithm, and each pixel is assigned to a cluster having the highest membership value. Experiment results show that the new algorithm is clearly superior to K-means and the adaptive algorithm. Furthermore, it is faster than the adaptive algorithm.

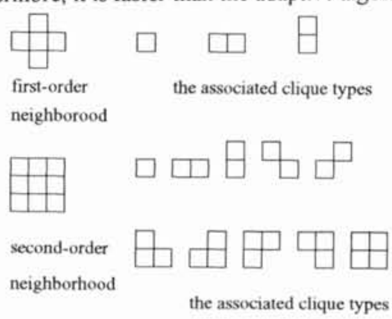

Fig.1 Clique types associated with first-order and second-order neighborhood system

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TABLE I

Computation time for K-means, the adaptive algorithm, and the proposed algorithm

Images	K-means algorithm	adaptive algorithm	proposed algorithm
Lena	11 s	20552 s	1756 s
House	8 s	22761 s	3703 s
F-16	10 s	16647 s	4423 s

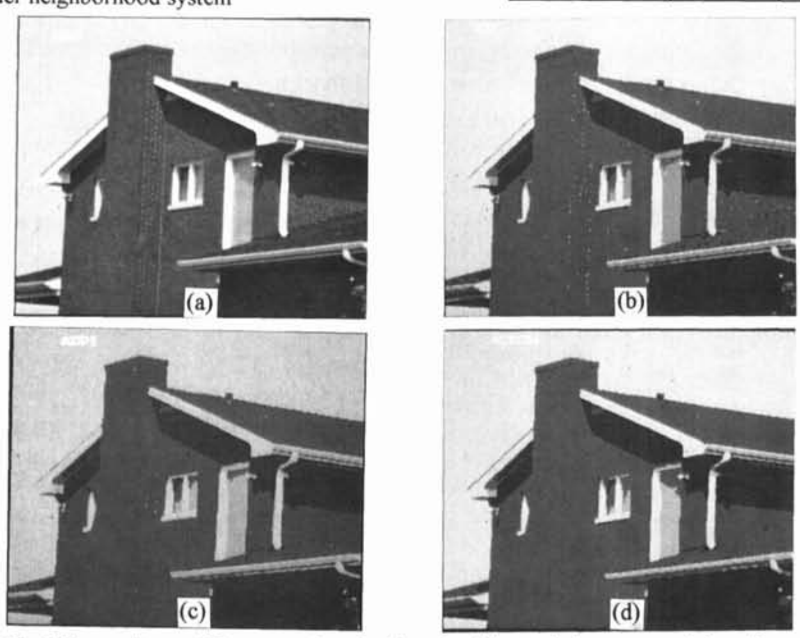

Fig.2 Comparison of K-means, the adaptive algorithm and the proposed algorithm of "House" (a) Original image. (b) Segmented image of K-means. (c) Segmented image of the adaptive algorithm. (d) Segmented image of the proposed algorithm.