MVA'94 IAPR Workshop on Machine Vision Applications Dec. 13-15, 1994, Kawasaki

COMPARING SOME TOOLS USING FREQUENCY DOMAIN FOR THE ESTIMATION OF 1-D (AND 2-D) DISPARITY

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ABSTRACT

In stereovision, the use of frequency domain methods raises the problems of the choice of a family of filters and a strategy to use it. This paper compares three families : Gabor filters, finite prolate spheroidal sequences and Weng's Windowed Fourier of Gaussian (WFG). Then a comparison between two strategies is done : an image-based choice of filters and the use of a "complete" subset of filters.

INTRODUCTION

Recently, the stereomatching problem has been addressed by the use of frequency domain methods in the form of phase differencing. The main idea comes from the Fourier shift theorem. In practice, disparity is local. Therefore, a **local** approach is used.

If we note $\phi_l(x)$ and $\phi_r(x)$ the phase of the output of complex banpass filters applied to left and right images, the disparity can be approximated by $d(x) = \frac{[\Phi_l(x)-\Phi_r(x)]_{2\pi}}{k(x)}$ where $-\pi < [\Phi]_{2\pi} \leq \pi$ and $k(x)$ is an estimate of the local frequency.

The good results gotten by many researchers confirm that these methods can be used as a first step towards a solution. However because poor results are obtained on areas which are not textured enough or with occluded regions, they have to be completed and (or) improved by other strategies. Nevertheless it is crucial to know both the intrinsic limits of the phase differencing method and the means to get the best disparity estimates.

These methods lead to two choices : which filter family and which strategy to use ? Once a family has been chosen, the next step is to know which filters within the family must be used. Fleet and other authors [1] suggested the following strategy : to choose the filters by using local image frequency contents. Calway et al. [2] proposed another one : to use a subset of filters, as complete as possible and chosen without considering the images.

Such a frequency domain approach leads to several underlying problems. The phase obtained after convolving the signal and the filter must be as linear as possible. When t˜he phase is not stable enough, singularities occur which have to be detected and dealt with.

This paper extends previous works like those of Westelius [3], Fleet and Jepson [4] or Jenkin and Jepson [5] on the comparison of filter families, emphasizing the experimental study. The main difference lies in our will to use the best filters possible within each family.

Multi-level random-dot stereograms [6] have been used. Some experiments have been carried out with stereograms whose texture was based on real images: the results were similar.

NOTATIONS

Among the possible filter families, three have been particularly studied. First, Gabor filters [7] verify the uncertainty relation [8] and thus are optimal. They are defined as

$$Gabor(x;\sigma,\omega) = g_\sigma(x)\, e^{i\omega x}, \qquad g_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \quad (1)$$

Finite prolate spheroidal sequences verify another optimal condition : when an index limited vector is bandlimited, the smallest energy is lost when this vector is a prolate. They can be generated by the following eigenvalue problem [9]

$$\begin{aligned} T_{n_1 n_2} B_{m_1 m_2} Prolate(n_1, n_2, m_1, m_1) = \\ \lambda\, Prolate(n_1, n_2, m_1, m_1) \end{aligned} \quad (2)$$

where

$$\begin{aligned} T_{n_1 n_2} &= (t_{n_1 n_2 kl})_{1\leq k,l\leq n} \\ t_{n_1 n_2 kl} &= \begin{cases} \delta_{kl} & n_1 \leq k \leq n_2 \\ 0 & k < n_1 \quad k > n_2 \end{cases} \\ B_{m_1 m_2} &= F^* T_{m_1 m_2} F \end{aligned} \quad (3)$$

with F being the DFT operator. Without loss of generality, we take $n_1 = 1+p$, $n_2 = 1-p$, $m_1 = 1$ ie, we only have two parameters (p and m_2). Like Calway [2], we use the prolate corresponding to the largest eigenvalue.

Weng [10] proposed a "Windowed Fourier of Gaussian" (WFG), the convolution between a windowed Fourier kernel and a Gaussian filter,

$$WFG(x;\sigma,\omega,M) = (h_{\omega,M} * g_\sigma)(x) \quad (4)$$

where

$$h_{\omega,M}(x) = \begin{cases} e^{i\omega x} & if\ |x| \leq \frac{M}{2} \\ 0 & otherwise \end{cases} \quad (5)$$

Following Weng, we use $M = \frac{2\pi}{\omega}, \sigma = \frac{M}{24}$ which leads to one parameter (ω).

SINGULARITIES

Fleet and Jepson [4] call singularity a point where filter response passes through the origin in the complex plane. It is an important cause of phase instability. Disparity measurements near a singularity may be very far from the true disparity. Fleet and Jepson give criterions to detect such points. The method is described here in one dimension.

If we note

$$\begin{aligned}(f * I_l)(x) &= a_l(x)e^{i\Phi_l(x)} \\ (f * I_r)(x) &= a_r(x)e^{i\Phi_r(x)}\end{aligned} \tag{6}$$

Then a point is considered as a singularity if it doesn't verify one of the constraints:

$$\begin{cases} \sigma\left|\Phi_l^{'}(x) - \omega\right| < \tau_\Phi \\ \sigma\left|\Phi_r^{'}(x) - \omega\right| < \tau_\Phi \end{cases} \quad \begin{cases} \sigma\frac{|a_l^{'}(x)|}{a_l} < \tau_a \\ \sigma\frac{|a_r^{'}(x)|}{a_r} < \tau_a \end{cases} \tag{7}$$

Choosing the thresholds in these criterions allows to approximately choose the percentage of points detected as singularities. Disparity measurements have been computed with the three families from the same stereogram with different thresholds. The results with a given Gabor filter wrt the threshold for the phase (τ_Φ) are presented in Fig. 1.

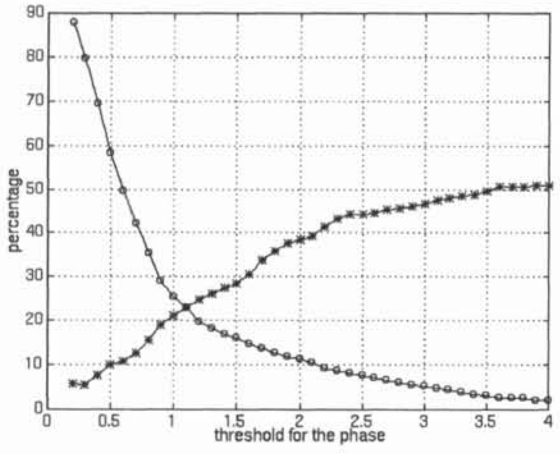

Fig. 1. Influence of threshold on singularity and disparity 1) * percentage of singularities 2) o percentage of wrong disparity estimates

To obtain good disparity measurements, a lot of points may have to be eliminated that way. If keeping as many points as possible is preferred, then the quality of measurement may be poor. Experiments show that values between 1 and 1.5 for the two thresholds are a good compromise (around 25 % of singularities). It is also worth noticing that most of the singularities are detected by the phase criterion.

COMPUTING OPTIMAL FILTERS

In this paper, and unless stated otherwise, the optimal filters are the filters which give the "best" results with the following algorithm :

1. convolving left and right images with the filter f;
2. computing the phase difference of the results gotten at step 1;
3. dividing this difference by an phase derivative $\phi'(x)$, approximated by using central finite difference
4. removing singularities;
5. interpolating in case of singularities;

The criterion is to minimize the RMS error or the percentage of wrong disparity measurements (disparity estimates are then rounded to the nearest integer .

Since prolates depend on integer parameters, the global optimization problem is difficult. It is often quicker, and safer, to test all possible prolate for a given vector dimension and to choose the best.

THE USE OF PHASE DERIVATIVE

Earlier methods [11] which estimate disparity from phase difference were based on dividing by the filter peak tuning frequency. Some authors prefer dividing by an approximation of phase derivative. In order to study the quantitative improvement, optimal filters in each family have been computed using the center frequency (or if needed, an approximation). Then, disparity has been estimated with these very filters, but with phase derivative. Finally, optimal filters with phase derivative approach have been computed. The criterions used were the percentage of wrong disparity estimates and the RMS error. We use in each case 100 multileval random-dot stereograms with two disparities (1 and 3). The results are showed in tables 1 and 2.

	Percentage of wrong disparity estimates		
	optimal filter (center frequency)	optimal filter but using phase derivative	optimal filter (phase derivative)
Gabor	15	13	8
WFG	55	36	34
prolate	32	24	17

Table 1. Improvement of phase derivative (wrong disparity estimates)

	RMS error		
	optimal filter (center frequency)	optimal filter but using phase derivative	optimal filter (phase derivative)
Gabor	7.8	5.9	5.0
WFG	12.7	10.7	9.7
prolate	13.9	10.8	9.0

Table 2. Improvement of phase derivative (RMS)

The use of phase derivative always improve the results even with filters optimal with the center frequency methods. We can note that Gabor filters give less errors than the other filters.

PHASE LINEARITY

In order to compare Gabor filters, prolate spheroidal sequences and WFG with respect to phase linearity, optimal filters in each family have been computed for a given random-dot stereogram. Of course, these optimal filters depend on the stereogram, the algorithm used to measure disparity and the criterion to choose the "best" filters. Here, the optimal filter is the one minimizing the percentage of wrong disparity measurements.

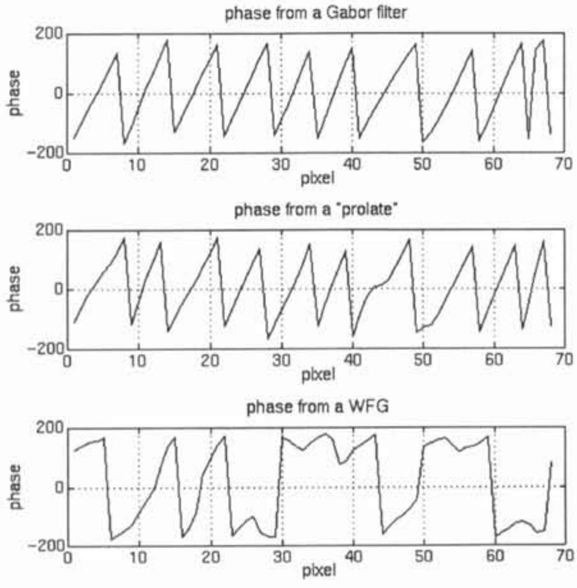

Fig. 2. Phase (in degrees) linearity

The results are presented in Fig. 2. Gabor filters give the most linear phase while the phase from the WFG is more irregular. Other algorithms and criterions have been used with consistent relative results.

INFLUENCE OF DISPARITY ON OPTIMAL FILTERS

We have computed optimal filters for each family for one multi-level random-dot stereogram in which two disparities were present, both varying from -3 to 3. We give here the best and the worst results, ie with Gabor filters and WFG, both using phase derivative method. The criterion consists on minimizing the percentage of wrong disparity estimates. Results are in tables 3 and 4.

	Gabor						
	-3	-2	-1	0	1	2	3
-3	0	0	5	14	20	7	16
-2	1	0	4	1	3	9	5
-1	4	0	0	1	3	5	4
0	6	4	1	0	4	3	5
1	14	7	9	1	0	0	16
2	6	7	8	4	0	0	1
3	13	13	13	8	6	1	0

Table 3. Percentage of wrong disparity estimates (Gabor)

	WFG						
	-3	-2	-1	0	1	2	3
-3	31	27	24	21	40	35	49
-2	25	3	13	21	23	43	23
-1	32	27	4	8	16	29	13
0	34	24	9	0	9	25	20
1	36	16	19	8	13	16	38
2	38	28	16	25	12	12	20
3	38	33	42	14	15	12	3

Table 4. Percentage of wrong disparity estimates (WFG)

Gabor filters and WFG do not have the same behavior. Optimal Gabor filters are very precise when there is few difference between the two disparities. Optimal WFG do not tolerate large disparity (in absolute value). We can see too that Gabor results are far better than WFG ones.

It is very difficult to characterize an optimal filter with respect to a given kind of stereogram. The quality of the results of a same filter can be very different when applied on two stereograms sharing the same disparity distribution.

So, in order to compare Gabor filters and WFG, we used 100 multi-level random-dot stereograms with two disparities (1 and 3). For each stereogram, we computed the optimal Gabor filter and the optimal WFG and we compared the respective results. The results are in Fig. 3

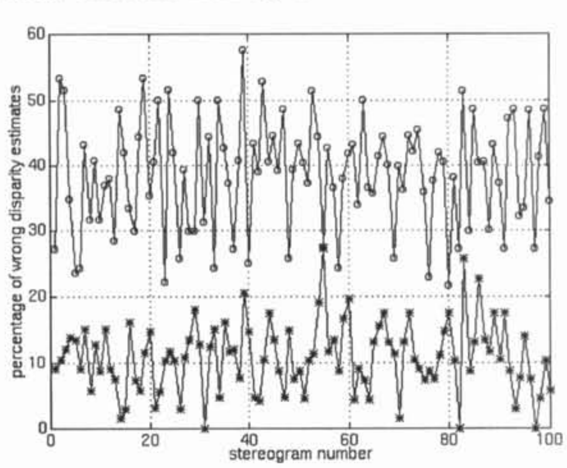

Fig. 3. Comparison between Gabor filters and WFG 1) * Gabor filters 2) o WFG

Gabor filters give better results than WFG. We can note that the results are rather unstable. We also have to add that the optimal Gabor filters bandwidths are often greater than one octave.

COMPARING FILTERS

A distance on filters is chosen so that we can say that two filters are "close to each other". We chose the quadratic norm. Close filters have been used and the disparity measurements compared. They were similar enough to validate the distance.

This distance allows the computing of the closest Gabor filter to any given prolate and the closest prolate to any given Gabor filter in order to compare similar filters. The conclusions of our experiments follow.

It is possible to compute a Gabor filter which is actually very close to a regular enough prolate. On the other hand, the closest prolate to a given Gabor filter can be quite far from the Gabor filter. The integer parameters of the prolates do not allow to be precise enough. A small change in a parameter can lead to a quite different prolate. But when a prolate and a Gabor filter close to each other can be found, they give similar results.

INFLUENCE OF INTEGER PARAMETERS

In order to compare the influence that have real and integer parameters, a large amount of experiments has been made.

Absolute results can be slightly unstable but relative results are consistent. Therefore, only qualitative results are given. For each stereogram, four filters have been tested :

1. the optimal Gabor filter;
2. the closest prolate to this Gabor filter;
3. the optimal prolate;
4. the closest Gabor filter to this prolate;

Whatever stereograms, criterions or algorithms used, relative results are the same. The optimal Gabor filter always gives the best disparity measurements. Then the optimal prolate and the closest Gabor filter give similar results. Finally, the closest prolate to the optimal Gabor filter give poor results.

This is consistent with what has been said earlier.

COMPARING METHODS

Calway et al. proposed a frequency domain method far different from classic phase difference. They use a subset, as complete as possible, of filters selected without considering the images. They call their algorithm the Multiresolution Fourier Transform (MFT). It seems interesting to compare the MFT with Fleet's [4] local frequency approach where filters are chosen by using local image frequency contents.

Both methods are independent of the family of filters. The only constraint lies in the local frequency approach. Since filters depend on the images, either every possible filter in the family is accessible before processing, or each filter can be easily and quickly generated as soon as it is needed by the algorithm. Gabor filters and WFG verify the latter condition.

With the MFT, since the filter subset generation does not require knowledge about the images, the complexity of the generation is not a problem. Calway et al use bandlimited prolate sequences defined through an eigenvalue problem, which is often time consuming.

To compare these two strategies, disparities have been recovered, first with Gabor filters and using local frequency and second with Calway et al's. method, computing the MFT and finding the peak of the local correlation between the two images. Following Calway et al., the filters are bandlimited prolate sequences. The results are shown in Fig. 4 and 5.

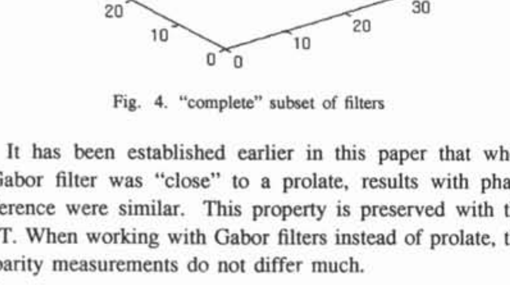

Fig. 4. "complete" subset of filters

It has been established earlier in this paper that when a Gabor filter was "close" to a prolate, results with phase difference were similar. This property is preserved with the MFT. When working with Gabor filters instead of prolate, the disparity measurements do not differ much.

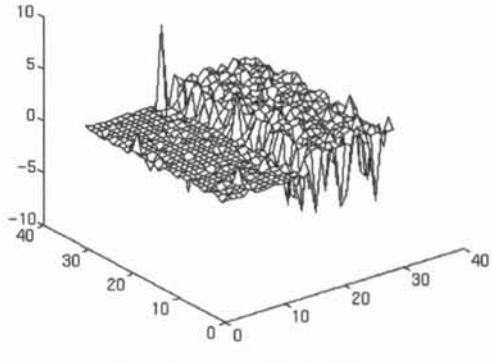

Fig. 5. image based choice of filters

CONCLUSION

This study comes from the need to use the most accurate tools to estimate disparity using phase differencing. Among the three families of filters compared here, the Gabor family appears to have the best behavior. As for the filter choice strategy, experiments show that filter subset based methods and image based methods give similar results. Further research includes the study of other families and a quantitative comparison of different improvements to image based methods (local frequency [4], Weng's iterative algorithm [10], ...).

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