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A REAL-TIME DEFORMABLE CLAY MODEL FOR ROTATIONALLY SYMMETRIC OBJECTS

Katsuyuki Kamei, Yasuaki Nakamura, and Shigeru Abe
Central Research Laboratory, Mitsubishi Electric Corporation

1-1, Tsukaguchi Honmachi, 8 Chome,
Amagasaki, Hyogo, 661 Japan

ABSTRACT

We have developed the interactive deformable clay modeler to provide a human designer with a method to interact with the virtual objects. In our model, the smoothness of the surface and the volume constancy are guaranteed even if any deformation is applied. Our method consists of two fundamental modeling concepts. One is the representation of the objects by the sliced cylinder model(SCM) and the other is smoothed contour model based on the Snake. The SCM determines the rough shape and the Snake makes the surface smooth.

1. INTRODUCTION

Recently, the modeling methods for representing the natural and non-functional shapes as well as the deformation of their shapes, called the deformable models, have been proposed. These methods are applied to several fields such as CG, computer vision and pattern recognition. Our research aims at the development of the computer supported conceptual design system for the 3D objects, using these deformable modeling methods.

Pentland and Williams[1] proposed the usage of the super-quadrics to represent the elastic deformable objects and applied it for the 3D object recognition. He also applied the modal dynamics to reduce the computational cost for the determination of the shapes. Terzopoulos and Fleischer[2] represented both the motion and the form of the objects by the new expressing method based on the theory of elasticity. He applied the method for the representation of the plastic clay as well. These methods, however, need more effort to realize real-time deformation.

A model proposed by Sato and Numazaki[3] can provide the interactive deformation of the plastic rotationally symmetric clay. In this model, to reduce computational time, the shape is represented by its 2-dimensional contour described by a small number of points. The modification operations can be applied just on one of those points at one time. Furthermore, the deformed shape is determined by only a few points near the actually modified point. The disadvantages of this method are that the number and the positions of those points affect the deformed shape, and that the smoothness of the contour(surface) is not considered at all.

We have developed the interactive deformable clay modeler which guarantees the smoothness of the surface and the volume constancy. Our method consists of two fundamental modeling concepts. One is the representation of the objects by the sliced cylinder model(SCM) and the other is the smoothed contour model based on the Snake. The former determines the rough shape and the latter makes the surface smooth. In both methods, the energy minimization techniques are employed.

2. VIRTUAL CLAY MODELER

2.1 Representation model of the clay

In our model, we assume the clay object has the following design constraints.

(1) A clay object has a symmetrical form which is given by rotating the contour line around the axis.

(2) The volume of the clay object does not change, even if its shape is deformed.

(3) The 2-D contour is continuous and differentiable (smooth).

The representation of the clay is shown in Fig.1. Let R be a rotationally symmetric object. R can be obtained by rotating its contour C,

$$C : v(u) = (X(u), Y(u)) \tag{1}$$

around y-axis, where $0 \le u \le 1$, $X(u) \ge 0$ and $Y(u) \ge 0$. For simplicity, hereinafter, we assume an object has no holes inside, that is, the inside of R is full with the clay. We also introduce the following constraints, in order to give fine operational environment to the users.

(4) The area to be modified by one action should not be limited to the narrow region.

(5) The rigidity of the clay can be varied by tuning some parameters.

(6) Deformations are performed in real-time.

2.2 Sliced cylinder model (SCM)

In our model, the object R is constructed by a collection of thin cylinders (Fig.2). Each of these cylinders can be represented by its radius (x value of the contour C) and thickness (difference of y value: dY). The shape of the object R is modified by changing the radius and the thickness of each cylinder under volume constancy. We define the energy functions corresponding to the changes of the shape and the position of the cylinders.

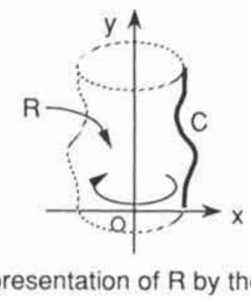


Fig.1 Representation of R by the contour C.

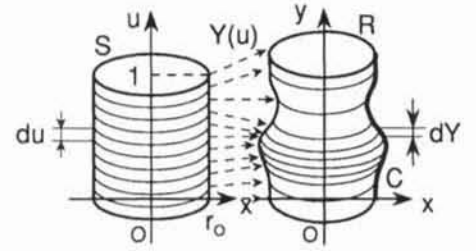


Fig.2 Sliced cylinder model.

Suppose that R is deformed from a cylinder S with a unit height and with radius r_0. Under volume constancy,

$$\pi r_0^2 du = \pi X^2 dY \tag{2}$$

$$\frac{dY}{du} = (\frac{r_0}{X})^2 \tag{3}$$

Therefore, dY/du decides the new radius X. The shape of R is represented by the Y(u). Here, we assume that Y(u) is increasing, a function of class C^2, and Y(0)=0.

Next, we define the evaluate function. Let $Y^0(u)$ and Y(u) be the shapes of R before and after deformation, respectively. First, for the change of the thickness of each sliced cylinder, dY - dY^0, we define the following strain energy.

$$E_s = \frac{1}{2}k(\frac{Y_u}{Y_u^0} - 1)^2 \qquad Y_u = \frac{dY}{du}, \quad Y_u^0 = \frac{dY^0}{du} \tag{4}$$

where k is a constant value. For the shift of each sliced cylinder, Y - Y^0, we define the following energy proportional to the shift.

$$E_m = \mu|Y - Y^0| \tag{5}$$

where μ is constant. The total energy with respect to the modification of the sliced cylinders is defined as

$$\begin{aligned} E_{move} &= \int_0^1 (E_s + E_m)du \\ &= \int_0^1 \{\frac{1}{2}k(\frac{Y_u}{Y_u^0} - 1)^2 + \mu|Y - Y^0|\}du \end{aligned} \tag{6}$$

We call this model "Sliced Cylinder Model", in short SCM.

2.3 Smooth surface by the Snake.

We introduce an energy minimization technique called Snake[4] to make the contour smooth. Snake is an active contour model for detecting or determining contour lines in images. The definition of the Snake is as follows. Energy is defined along a curve, and the contour is determined as the curve which minimizes this energy. This energy is defined by the internal energy E_{int}, the external constraint energy E_{con}, and the image energy E_{img}. Let a contour be v(u) = (x(u),y(u)), 0≤u≤1. The total energy is

$$E_{snake} = \int_0^1 (E_{int} + E_{con} + E_{img})du \tag{7}$$

E_{int} operates so that the contour gets smooth. We use the following E_{int} to define the smoothing energy of our model,

$$E_{smooth} = \int_0^1 E_{int}du = \int_0^1 \{\alpha|v_u|^2 + \beta|v_{uu}|^2\}du \qquad v_u = \frac{dv}{du}, \quad v_{uu} = \frac{d^2v}{du^2} \tag{8}$$

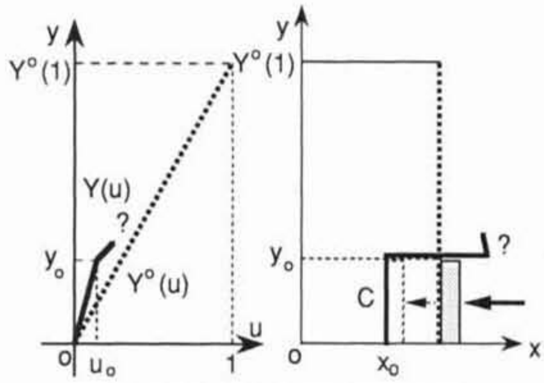


Fig.3 The given modifying operation.
The contour C (right) and its Y(u) (left).

where α and β are parameters which control the properties of the contour.

3. DEFORMATION BY ENERGY MINIMIZATION

3-1 Rough representation obtained by SCM

The total deformation energy of R is defined as follows by E_{move} and E_{smooth}.

$$\begin{aligned} E_{total} &= E_{move} + E_{smooth} \\ &= \int_0^1 \{\frac{1}{2}k(\frac{Y_u}{Y_u^0} - 1)^2 + \mu|Y - Y^0|\}du \\ &\quad + \int_0^1 \{\alpha|v_u|^2 + \beta|v_{uu}|^2\}du \end{aligned} \tag{9}$$

We minimize this energy under the condition of volume constancy (3). We employ 2 step minimization. In the first step, E_{move} is minimized and the rough deformed shape is obtained. In the second step, E_{total} is minimized by the Greedy algorithm[5] using the rough shape as the initial shape of its iteration.

The detail of how to deform in the first step is described below. Some part of the contour C is modified externally, such as squashing its $0 \le y \le y_0$ part to x_0, as shown in Fig.3. Where Y(u)>0, X(u)>0, Y(0)=0 and the clay should be moved to the area of $y > y_0$. In the area of $0 \le y \le y_0$, the deformed shape Y(u) is determined by this squashing,

$$Y(u) = (\frac{r_0}{x_0})^2 u \tag{10}$$

Let u_0 be a value which satisfies $Y(u_0) = y_0$,

$$u_0 = (\frac{x_0}{r_0})^2 y_0 \tag{11}$$

In the area of $y_0 \le y (u_0 \le u \le 1)$, the Euler's equation is

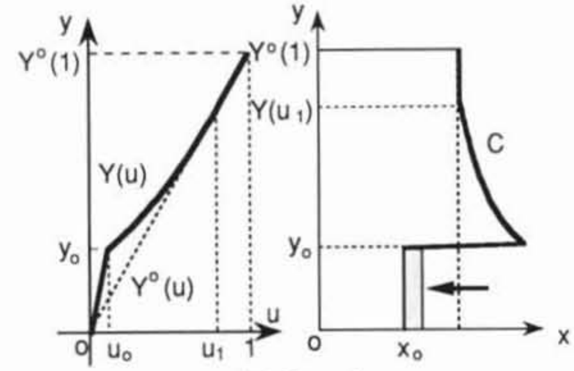
$$\mu - \frac{d}{du}\{\frac{k}{Y_u^0}(\frac{Y_u}{Y_u^0} - 1)\} = 0 \tag{12}$$

Here we must consider the boundary conditions for solving equation (12). There are three sets of the conditions according to the types of the deformation. The first type is the case that the deformation is occurred locally in the area of $u_0 \le u \le u_1 (<1)$ (Case I). In this case, $Y(u) = Y^0(u)$ holds in the area $u > u_1$. The boundary conditions of this case are

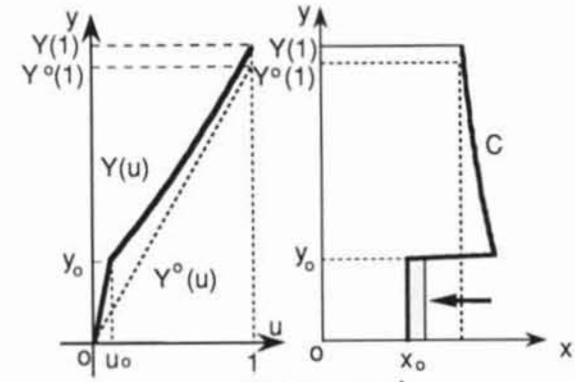
$$Y(u_0) = y_0, \quad Y(u_1) = Y^0(u_1), \quad Y_u(u_1) = Y_u^0(u_1) \tag{13}$$

u_1 satisfies

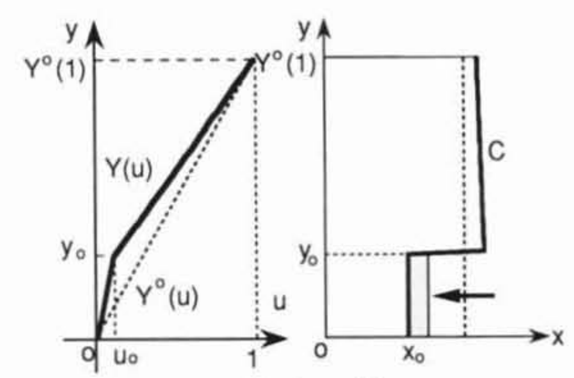
$$\frac{k}{\mu}\{y_0 - Y^0(u_0)\} = \int_{u_0}^{u_1} (u_1 - u)(Y_u^0)^2 du \tag{14}$$



(a) Case I.



(b) Case II.



(c) Case III.

Fig.4 The resultant shapes by the sliced cylinder model.

We obtain u_1 by solving (14) numerically. Then, we get the deformed shape with u_1.

$$Y(u) = \frac{\mu}{k}\int_{u_0}^{u}(u-u_1)(Y_u^0)^2du + Y^0(u) - Y^0(u_0) + y_0 \quad (15)$$

In this case, u_1 must be between u_0 and 1.

When u_1 becomes larger than 1, deformation should be occurred in the entire area of $u_0 \leq u$ (≤ 1). The second case is that Y(1) is possible to be larger than $Y^0(1)$ (Case II), such that u=1 means the top of the clay. The boundary conditions of Case II are

$$Y(u_0) = y_0 \ , \qquad Y_u(1) = Y_u^0(1) \quad (16)$$

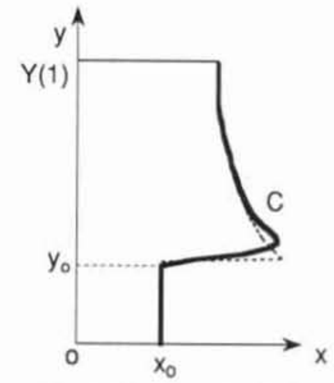


Fig.5 The resultant shape by minimizing E_{total}.

The third type is that Y(1) is fixed to $Y^0(1)$ (Case III), where u=1 means the bottom of the clay. The boundary conditions of Case III are

$$Y(u_0) = y_0 \ , \qquad Y(1) = Y^0(1) \quad (17)$$

We obtain the following equations (18) and (19) for Case II and Case III, respectively.

$$Y(u) = \frac{\mu}{k}\int_{u_0}^{u}(u-1)(Y_u^0)^2du + Y^0(u) - Y^0(u_0) + y_0 \quad (18)$$

$$Y(u) = \frac{\mu}{k}\int_{u_0}^{u}(u-c)(Y_u^0)^2du + Y^0(u) - Y^0(u_0) + y_0 \quad (19)$$

$$c = \frac{\frac{k}{\mu}(y_0 - Y^0(u_0)) + \int_{u_0}^{1} u(Y_u^0)^2du}{\int_{u_0}^{1}(Y_u^0)^2du}$$

Fig.4 shows examples of these three cases. When the modification is operated to the central part of R, we use the above methods independently on the upper and the lower parts of the modified area.

In this method, the deformed shape and the area can be changed by tuning the parameter μ/k. In the examples shown in Fig.4, μ/k is 1.0 in Case I, and 0.1 in Case II and Case III.

3-2 Determination of the final deformed shape

In the second step of our method, we get the final shape by minimizing E_{total} by the Greedy algorithm. We use the rough shape obtained in the first step as the initial shape. We discretize the contour of the rough shape

$$C : v_i = (X_i, Y_i), \quad X_i = X(ih), \quad Y_i = Y(ih) \quad (20)$$
$$i = 0,1,..,n \qquad nh = 1$$

and

$$Y_{ui} \approx \frac{Y_{i+1} - Y_i}{h}, \ v_{ui} \approx (\frac{X_{i+1} - X_i}{h}, \frac{Y_{i+1} - Y_i}{h})$$
$$v_{uui} \approx (\frac{X_{i+1} - 2X_i + X_{i-1}}{h^2}, \frac{Y_{i+1} - 2Y_i + Y_{i-1}}{h^2}) \quad (21)$$

Volume constancy (3) is represented,

$$\frac{Y_{i+1} - Y_i}{h} = (\frac{r_0}{X_i})^2 \quad (22)$$

Then, Y_i of v_i is moved in its neighbour if the value of E_{total} is decreased by the move, and the coordinates of v_i and v_{i+1} are also changed according to the move Y_i. This process is applied sequentially and iteratively to all points v_i except those which are squashed externally, and terminated

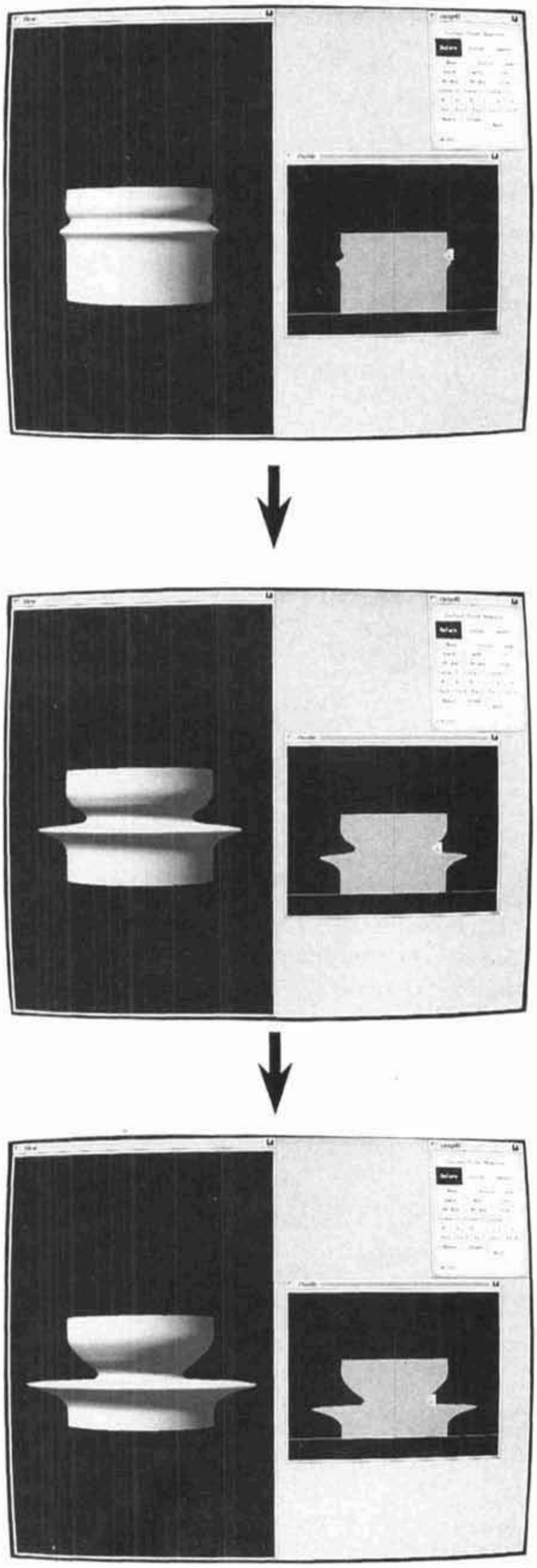


Fig.6 Deformation when squashing downward.

Fig.7 An example of shape design.

if the number of the moved points becomes less than the threshold value. Then, we get the final deformed shape constructed by v_i. Fig.5 shows an example of applying the second step to the contour shown in Fig.4(a), where $\alpha=1.0$, $\beta=1.0$, $\mu=1.0$ and $k=1.0$.

4. EXAMPLES AND RESULTS

Some examples of deformation are shown in Fig.6 and Fig.7. Fig.6 indicates the deformation of the object R when the external squashing operation is given downward in succession. The operation is shown by the move of the pentagon in the figures. In this example, the contour is discretized by 64 points, $\alpha=1.0$, $\beta=1.0$, $\mu=1.0$ and $k=1.0$. The time taken to get and display the deformed shape is about 0.3 seconds on Personal Iris 4D/35TG workstation. Fig.7 shows an example of a created shape using our clay modeler.

Our method satisfies the conditions (4) - (6) described in Chapter 2: the deformed area is represented by u_1 which is different for each deforming action, the rigidity of the clay can be tuned by μ and k, and the response is quick enough for interactive operations.

5. CONCLUSION

We have developed the interactive deformable clay modeler. Our modeling method consists of two fundamental modeling concepts of the sliced cylinder model(SCM) and the smoothed contour model based on the Snakes, and employs the energy minimization techniques. The features of our method are that a shape can be deformed in succession, the rigidity of the clay can be tuned, and the response is quick enough for interactive operations.

We will extend this model for general 3D objects and for a system which makes it possible for a designer to create desired shapes more efficiently.

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