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INVESTIGATION ON CALIBRATION PROBLEM

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ABSTRACT

In this paper, we treat the calibraition problem in terms of projective geometry. The projective essences of camera calibration and calibrating active 3D vision system composed of a camera and a plane structured light source are discussed. And based on the theorems in projective geometry, the necessary and sufficient conditions for solving these calibration problems have been given and proved. By linear projective geometrical model and a series of meaningful homogeneous transformations, a new calibration algorithm for 3D vision system is proposed. An accuracy of 0.1mm at a distance of 400mm in 3D measurement has been reached, and with the necessary devices, the proposed calibration procedure is full automatic and can be done in real time.

KEYWORDS : camera calibration, calibration problem, 3D vision system

1. INTRODUCTION

Calibration of 3D vision system consists of camera calibration and auxiliary-device (e.g. strutured light source in an active vision system) calibration. It has been generally accepted that camera calibration is the process of determining the camera internal geometric and optical characteristices and/or the 3D position and orientation of the camera frame relative to a certain world coordinate system. Auxiliary-device calibration means the determination of the 3D position and orientation of the auxiliary-device relative to the world coordinate system.

Calibration of 3D vision system has been paid much attention for a long term, and there have been developed many practical algorithms and techniques in different applications. [4]-[17] In this paper, the calibration problem will be looked into in terms of projective geometry. At first, in section 2, we will describe and prove the neccessary and sufficient condition of determiningg the 2D perspectivity based on theorems in projective geometry. Then the projective geometry essences of calibrating single camera and an active 3D vision system with structured plane light will be shown up. Several calibration algorithms published in recent years will also be reviewed with the emphysis on analysing the geometrical essences of them and discussing their necessary conditions for unique calibration results. In section 3, a new calibration algorithm will be proposed based on linear projective geometrical model. The algorithm has the advantage of that the final calibration equations can be reached through a series of meaningful homogeneous transformation. It can be done in real time with necessary devices(mainly a 3D micrometer stage) and meet a high accuracy of 0.1mm at a distance of 400mm in 3D measurement . Although our algorithm is mainly designed to calibrate an active 3D vision system with a plane structured light source, it can also be used in the general cases of single camera calibration. Finally, in section 4, the calibration procedure and experiments will be described briefly and the factors which may cause the errors in calibration results will be discussed. To minimize the errors, an error probability model is set and weighed least square method is employed. Some corresponding results will also be given here.

2. THE PROJECTIVE GEOMETRY ESSENCE OF 3D VISION SYSTEMS CALIBRATION AND REVIEW ON SEVERAL CALIBRATION ALGORITHMS OR TECHNIQUES

2.1. Projectivity, Perspectivity and Foundamental Theorem of Perspectivity

Here, we would extend every concerned Cartesian line and plane to projective line and plane, respectively, i.e. problems would be considered and solved absolutely in terms of projetive geometry. For the sake of clearness, we transcribe the definitions of the key notions to be used. [2] [3]

. 2D Projectivity : For a point set (x), point x has the coordinates (x1, x2, x3) relative to a certain projective coordinate system, and another point set (x'), point x' has the coordinates (x1', x2', x3') relative to a certain projective coordinate system, if there is a 3×3 matrix ψ, $|\psi| \neq 0$, it leads:

$$p \begin{bmatrix} x1' \\ x2' \\ x3' \end{bmatrix} = \psi \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} ,$$

(p can be various with different x)
then the mapping from (x) to (x') will be one to one, and called projectivity.

. 2D Perspectivity : Given a pencil g(n) and two different planes π , π', $g(n) \neq \pi$, π', as Fig. 1 shows. The mapping between the intersections of g(n) and π, π', respectively, is called perspectivity. And the fixed point g of the pencil g(n) is known as perpective center.

It could be proved that a (2D) perspectivity, for certain projective coordinates systems, is just a (2D) projectivity. (See in Appendix I) In projective geometry, there is the foundamental theorem of 2D projectivity as follows :

There is exactly one projectivity which maps, in a specified order, a given quadrangular set within plane π onto another given quadrangular set within plane π'

Based on the above theorem, we could have the foundamental theorem of 2D perspectivity as follows :

For certain projective coordinate systems, there is exactly one perspectivity which maps, in a specified order, a given quadrangular set within plane π onto another given quadrangular set within plane π'.

For the proof detail, please see Appendix I.

2. 2. The Projective Geometry Essence of Single Camera Calibration

Single camera calibration is important in robot vision research, especially for 3D vision systems based on stereo vision or motion vision. As we show in section 1, camera calibration involves the determination of the camera internal geometrical and optical characteristics and/or the 3D position and orientation of the camera frame relative to a certain world coordinate system. Here, we put our emphasis on the external spatial parameters calibration. For camera intrisinc parameters calibration, please refer to Tsai, R.Y. and his collegues' works. [8] [9]

By the assumption of having no nonlinear distortion, the geometrical model of a camera can simply be composed of camera image plane π_I and lens center Oc, i.e. the pin-hole model is selected. In addition, there are the selected camera coordinate system and external reference coordinate system, see Fig. 1.

Because the origin of the selected camera coordinate system [Oc] is located just at the lens center and there exist relations between [Oc] and camera frame coordinate system [O_F] as : $\overline{X_C} // \overline{X_F}$, $\overline{Y_C} // \overline{Y_F}$, $\overline{Z_C} // \overline{X_C} \times \overline{Y_C}$, the camera coordinate system [Oc] can denote the position and orientation of the camera frame.

If we selected a plane π_R which did not pass through the lens center Oc, then the mapping of the point set (x) on π_R and their image point set (x') on π_I is a 2D perspectivity, and Oc is the perspective center. For a selected 2D image coordinate system [O_F] on image plane π_I and a 3D external reference coordinate system [O_R], to exactly determine the perspectivity between π_R and π_I only one pair of corresponding quadrangular sets are required, from the conclusion in section 2.1. From the definitions and proof procedures in Appendix I, we know that metrices [M1], [M2] are stationary relative to the camera and matrix [M4'] is just determined by the selected reference plane . Because of the equation $[\psi]=[M1]\cdot[M2]\cdot[M3]\cdot[M4']$, exactly determining the perspectivity $[\psi]$ will lead to the unique determination of matrix [M3]. [M3] is the homogeneous transformation matrix between the camera coordinate system and the external referance coordinate system. So, we know that only one pair of corresponding quadrangular sets are required to exactly determine the position and orientation of the camera frame relative to the external referance coordinate system . This is the theoretic conclusion of camera calibration problem. Fischler, M. and Bolles, R. [17] have decribed how to determine the camera position and orientation with four coplannar control points(no three of which are collinear) and their images, the formula solution has also been given. With three control points, they have gotten more than one group solution. Namely, their results measure up to the above theoretic conclusion we have drawn.

2.3. Projective Essence of Calibrating An Active 3D Vision System with A Structured Plane Light

Again without considering nonlinear distortion, the geometrical model of an active 3D vision system with structured plane light is composed of light plane π_L, image plane π_I and a fixed point Oc(lens center), see Fig. 2. For camera, pin-hole model is employed here again.

The mapping between the point set (x) on light plane π_L and their image point set (x') on image plane π_I is a 2D perspectivity, the perspective center is Oc. In Fig. 2, [Oc] is a selected camera coordinate system, [O_R] is a selected external reference coordinate system and [O_F] is the image coordinate system.

The goal of calibrating this kind of vision systems is to determine the exact mapping relation between the point on light plane and its image point on image plane [5] [7]. If the relation has been known, we will be able to calculate the 3D coordinates of a spatial point from its image position. Based on the foundamental theorem of perspectivity in section 2.1, this mapping relation can be exactly determined by a pair of cooresponding quadrangular sets on π_L and π_I

. Of course, this is just a theoretic result without considering errors. In practice, a pair of cooresponding quadrangular sets generally can not meet the desired accuracy. Investigators always design their calibration algorithms which can employ more cooresponding points, or cooresponding edges with numerical processing methods, e.g. least-square, trying to reduce errors. But using fewer cooresponding points as far as it could maintain the accuracy is still what the most desired.

2.4. Review on Several Calibration Algorithms or Techniques

Yakimovsky, Y. and Cunningham, R. proposed a calibration algorithm in 1978 for their 3D vision system which employed two TV cameras based on stereo vision. The algorithm was designed for single camera calibration, i.e. the 3D position and orientation of the camera frame relative to a certain 3D coordinate system could be known after calibration. Pin-hole camera model was selected and perspective transformation was computed without the consideration of nonlinear errors(e.g. lens distortion). To get the unique calibration result of a camera, at least eight sample points and their images are needed. An accuracy of ±5mm at a distance of 2m in 3D measurement is reported. [4]

Agin, G. J. and Highnam, P. T. developed a calibration techenique for their Eye-in-hand system in 1982. The vision system, which was mounted on the wrist of a manipulator, consisted of a camera and a plane light source. Again, pin-hole camera model and homogeneous transformation are used in their paper. There are fourteen unknown parameters(six for the camera, six for the light source) needed to calibrate. The proposed technique calibrated these parameters separately. They reported that the overall accuracy at a working distance of 30mm would be in the neihborhood of 3mm. [5]

Isaguire, A., Pu, P. and Summers, J. presented the related work for camera calibration in 1985. In their approach, two-planes camera model and polynomial interpolation were proposed. The needed sample pairs dependent on the polynomial degree. For second degree, at least six sample pairs needed to exactly determine the unknown coeffients. For third degree, ten pairs needed. Generally, (n+1)(n+2)/2 sample pairs will be needed for nth polynomial degree. No experiment result was reported. [6]

Chen, C. H. and Kak, A. C. proposed a calibration algorithm in 1987 for an active 3D vision system with a structured plane light. Pin-hole camera model was selected. No nonlinear errors was considered in their algorithm. The algorithm was developed from the concerned conclusion of the cross ratio in projective geometry. Four coplannar points, no three of which are colinear, and their images can exactly determine the calibration result. But the article proposed that six different sample edges and their images can meet higer accurate result. An accuracy of 0.8mm at a distance of 200mm, 8mm at a distance of 500mm was reported. [7]

Tsai, R. Y. developed a two-stage camera calibration technique in 1987. The proposed calibration algorithm pays much attention to the lens distortion and other intrinsic optical characteristics of cameras. Radial lens distortion model was selected. The paper reported that the maximum error at a distance of 100mm would be 0.05mm while the average one would be 0.015mm. [8] [9]

3. A NEW CALIBRATION ALGORITHM FOR 3D VISION SYSTEMS

In this section, we would present a calibration algorithm which is mainly considered to serve for active 3D vision systems with structured plane lights. The following inference and experiment results show that this algorithm can determine the mapping between the image plane and the structured light plane of such an active vision system accurately and effectively. Furthermore it can calibrate the intrinsic characteristics and the 3D position and orientation of the camera frame relative to a certain external reference coodinate system. Besides it can also derive the 3D equation of the structured light plane relative to the reference coodinate system. After calibration, the 3D external coordinates of points on structured light plane can be infered from their images positions in image frame.

The purpose of employing active 3D systems is for getting the high accuracy 3D information of object surfaces, its applications are inspection and/or assembly of industrial parts. So we can assume that the observed objects are located in a limited range relative to the camera and narrow angle lens is used. Based on these assumption, the camera geometrical model can be selected as pin-hole model without consideration of nonlinear distortion and the whole inference can be executed thoroughly in terms of homogeneuos coodinates and homogeneuos transformation.

3.1. Geometrical Model and Spatial Relations

Based on the linear assumption, the geometrical model of an active 3D vision system with a plane light can be set up with the selected coodinate systems as Fig.3 shows. For camera, pin-hole model is adopted here.

One point should be noted that the camera coordinate system $[O_F]$ here is the 2D coordinate system on the frame emerged by computer sampling. In another word, the factors such as horizontal scale factor introduced by Tsai, R.Y. in [8] [9] have been taken account into imaging procedure. The selected camera coordinate system $[O_C]$ meets: $\overline{X_C}//\overline{X}$, $\overline{Y_C}//\overline{Y}$, $\overline{Z_C}//\overline{X_C}\times\overline{Y_C}$; and coodinate systems $[O_F]$, $[O_C]$ and $[O_R]$ are all orthogonal ones. In the following, we would list and infer the related spatial relations.

Firstly, there is percpective transformation as follows:

$$\begin{bmatrix} \Delta x_F \\ \Delta y_F \\ 1 \end{bmatrix} = (1/H)\cdot\begin{bmatrix} -1/r_x & 0 & 0 & 0 \\ 0 & -1/r_y & 0 & 0 \\ 0 & 0 & 1/r_z & 0 \end{bmatrix}\cdot\begin{bmatrix} x_C \\ y_C \\ z_C \\ 1 \end{bmatrix} \qquad (1)$$

$$\Delta x_F = x_F - x_F(O_I)\ ; \qquad \Delta y_F = y_F - y_F(O_I)$$

where r_x and r_y are the spacing between pixels in computer image frame on X_F and Y_F directions, respecticely. r_z is the distance from lens center to image plane, i.e. $||\overline{O_C O_I}||$.

Secondly, for coordinate systems $[O_C]$ and $[O_R]$, there is homogeneous transformation:

$$\begin{bmatrix} x_C \\ y_C \\ z_C \\ 1 \end{bmatrix} = \begin{bmatrix} a_x^{RC} & b_x^{RC} & c_x^{RC} & d_x^{RC} \\ a_y^{RC} & b_y^{RC} & c_y^{RC} & d_y^{RC} \\ a_z^{RC} & b_z^{RC} & c_z^{RC} & d_z^{RC} \\ 0 & 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} x_R \\ y_R \\ z_R \\ 1 \end{bmatrix} \qquad (2)$$

where the meaning of every symbol is listed in Appendix II.

Hence, we could get the mapping from 3D reference coordinate system $[O_R]$ to 2D image coordinate system $[O_F]$ as follows:

$$\begin{bmatrix} \Delta x_F \\ \Delta y_F \\ 1 \end{bmatrix} = (1/H)\begin{bmatrix} -a_x^{RC}/r_x & -b_x^{RC}/r_x & -c_x^{RC}/r_x & -d_x^{RC}/r_x \\ -a_y^{RC}/r_y & -b_y^{RC}/r_y & -c_y^{RC}/r_y & -d_y^{RC}/r_y \\ a_z^{RC}/r_z & b_z^{RC}/r_z & c_z^{RC}/r_z & d_z^{RC}/r_z \end{bmatrix}\cdot\begin{bmatrix} x_R \\ y_R \\ z_R \\ 1 \end{bmatrix} \qquad (3)$$

where

$$H = (a_z^{RC}x_R + b_z^{RC}y_R + c_z^{RC}z_R + d_z^{RC})/r_z$$

Determing the mapping from 2D image coordinate system $[O_F]$ to 3D reference coordinate system $[O_R]$, there needs extra constraints. Here we have the structured light plane constraint, i.e. for every point on the light plane, there is:

$$\begin{bmatrix} x_R \\ y_R \\ z_R \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ k_x & k_y & k_o \\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} x_R \\ y_R \\ 1 \end{bmatrix} \qquad (4)$$

From (3) and (4), it can be known:

$$\begin{bmatrix} \Delta x_F \\ \Delta y_F \\ 1 \end{bmatrix} = (1/H)\begin{bmatrix} -(a_x^{RC}+k_x c_x^{RC})/r_x & -(b_x^{RC}+k_y c_x^{RC})/r_x & -(d_x^{RC}+k_o c_x^{RC})/r_x \\ -(a_y^{RC}+k_x c_y^{RC})/r_y & -(b_y^{RC}+k_y c_y^{RC})/r_y & -(d_y^{RC}+k_o c_y^{RC})/r_y \\ (a_z^{RC}+k_x c_z^{RC})/r_z & (b_z^{RC}+k_y c_z^{RC})/r_z & (d_z^{RC}+k_o c_z^{RC})/r_z \end{bmatrix}\begin{bmatrix} x_R \\ y_R \\ 1 \end{bmatrix}$$

For the clarity in the following inference, we denote the above formula as follows:

$$\begin{bmatrix} \Delta x_F \\ \Delta y_F \\ 1 \end{bmatrix} = (1/H)\cdot\begin{bmatrix} N \end{bmatrix}\cdot\begin{bmatrix} x_R \\ y_R \\ 1 \end{bmatrix} \qquad (5)$$

where $H = N(3,1)x_R + N(3,2)y_R + N(3,3)$, $N(i, j)$ denotes the element of matrix $[N]$ at the position of the ith row and the jth colomn.

From (5), we can get the solutions of x_R and y_R expressed by $N(i, j)$, Δx_F and Δy_F. Then combining them with (4), we could derive the mapping from 2D image coordinate system $[O_F]$ to 3D reference coordinate system $[O_R]$ as follows:

$$\begin{bmatrix} x_R \\ y_R \\ z_R \\ 1 \end{bmatrix} = (1/H')\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ k_x & k_y & k_o \\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} M \end{bmatrix}\cdot\begin{bmatrix} \Delta x_F \\ \Delta y_F \\ 1 \end{bmatrix} \qquad (6)$$

where:

$$\begin{bmatrix} M \end{bmatrix} = \begin{bmatrix} (N(2,3)N(3,2)-N(2,2)N(3,3)) & (N(1,2)N(3,3)-N(1,3)N(3,2)) & (N(1,3)N(2,2)-N(1,2)N(2,3)) \\ (N(2,1)N(3,3)-N(2,3)N(3,1)) & (N(1,3)N(3,1)-N(1,1)N(3,3)) & (N(1,1)N(2,3)-N(1,3)N(2,1)) \\ (N(2,1)N(3,2)-N(2,2)N(3,1)) & (N(1,2)N(3,1)-N(1,1)N(3,2)) & (N(1,1)N(2,2)-N(1,2)N(2,1)) \end{bmatrix}$$

$$H' = (N(2,1)N(3,2)-N(2,2)N(3,1))\Delta x_F + (N(1,2)N(3,1)-N(1,1)N(3,2))\Delta y_F + (N(1,1)N(2,2)-N(1,2)N(2,1))$$

3.2. Equations for Parameters Calibration

3.2.1. For Mapping between the Image Plane and the Structured Light Plane

In active 3D vision systems, mostly the purpose of calibration is to be able to infer 3D external coordinates from their 2D image coordinates. In the inference in section 3.1. , there are several equations imply the relation which can meet this purpose. But for the sake of computional convenience, formula (5) has been selected.

The Δx_F and Δy_F in (5) should be calculated based on image center $(x_F(O_I), y_F(O_I))$, but the accurate position of the center is not known in general cases. To avoid direct use of the center's coordinates, here a further inference is done. It is known that :

$$\begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_p(O_I) \\ 0 & 1 & y_p(O_I) \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta x_p \\ \Delta y_p \\ 1 \end{bmatrix} \triangleq \begin{bmatrix} N_1 \end{bmatrix} \cdot \begin{bmatrix} \Delta x_p \\ \Delta y_p \\ 1 \end{bmatrix}$$

Combining with (5), there is :

$$\begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} N_1 \end{bmatrix} \cdot (1/H) \cdot \begin{bmatrix} N \end{bmatrix} \begin{bmatrix} x_R \\ y_R \\ 1 \end{bmatrix} \triangleq (1/H) \cdot \begin{bmatrix} N' \end{bmatrix} \cdot \begin{bmatrix} x_R \\ y_R \\ 1 \end{bmatrix}$$

where $H = N'(3,1)x_R + N'(3,2)y_R + N'(3,3)$.

Based on this relation, the calibration equations can be writen as follows:

$$-\frac{N'(1,1)}{N'(1,3)}x_R - \frac{N'(1,2)}{N'(1,3)}y_R + \frac{N'(3,1)}{N'(1,3)}x_R \cdot x_p + \frac{N'(3,2)}{N'(1,3)}y_R \cdot x_p + \frac{N'(3,3)}{N'(1,3)}x_p = 1$$

$$-\frac{N'(2,1)}{N'(2,3)}x_R - \frac{N'(2,2)}{N'(2,3)}y_R + \frac{N'(3,1)}{N'(2,3)}x_R \cdot y_p + \frac{N'(3,2)}{N'(2,3)}y_R \cdot y_p + \frac{N'(3,3)}{N'(2,3)}y_p = 1$$

We denote them in abbreviation:

$$(-x_R)n_{11} + (-y_R)n_{12} + (x_R \cdot x_p)n_{13} + (y_R \cdot x_p)n_{14} + (x_p)n_{15} = 1$$

$$(-x_R)n_{21} + (-y_R)n_{22} + (x_R \cdot y_p)n_{23} + (y_R \cdot y_p)n_{24} + (y_p)n_{25} = 1$$

There are eight independent unknown parameters in these two equations. To exactly determine them, eight independent equations are needed and also enough. That means knowing four point samples, no three of which are collinear, on the structured light plane and their images can lead a unique calibration result. This is the necessary condition of our calibration algorithm; it is exactly consistent with the theoritic conclusion in section 2.3..

3.2.2. For the Equation of the Structured Light Plane

From (4) in section 3.1. , the calibration equation for the structured light plane can be writen as follows:

$$\left(\frac{x_R}{z_R}\right)k_x + \left(\frac{y_R}{z_R}\right)k_y + \left(\frac{1}{z_R}\right)k_o = 1$$

To exactly determine the three coefficients k_x, k_y and k_o in the equation, three sample points, which are not collinear, on the structured light plane are needed and also enough without considering errors.

3.2.3. For Single Camera Calibration

From (3) in section 3.1. , the calibration equations for the camera can be writen as follows:

$$\left(\frac{a_x}{d_x}\right)x_R + \left(\frac{b_x}{d_x}\right)y_R + \left(\frac{c_x}{d_x}\right)z_R + \left(\frac{a_z}{d_x} \cdot \frac{r_x}{r_z}\right)x_R \cdot \Delta x_p + \left(\frac{b_z}{d_x} \cdot \frac{r_x}{r_z}\right)y_R \cdot \Delta x_p + \left(\frac{c_z}{d_x} \cdot \frac{r_x}{r_z}\right)z_R \cdot \Delta x_p + \left(\frac{d_z}{d_x} \cdot \frac{r_x}{r_z}\right)\Delta x_p = -1$$

$$\left(\frac{a_y}{d_y}\right)x_R + \left(\frac{b_y}{d_y}\right)y_R + \left(\frac{c_y}{d_y}\right)z_R + \left(\frac{a_z}{d_y} \cdot \frac{r_y}{r_z}\right)x_R \cdot \Delta y_p + \left(\frac{b_z}{d_y} \cdot \frac{r_y}{r_z}\right)y_R \cdot \Delta y_p + \left(\frac{c_z}{d_y} \cdot \frac{r_y}{r_z}\right)z_R \cdot \Delta y_p + \left(\frac{d_z}{d_y} \cdot \frac{r_y}{r_z}\right)\Delta y_p = -1$$

We denote them in another way:

$$(x_R)m_{11} + (y_R)m_{12} + (z_R)m_{13} + (x_R \cdot \Delta x_p)m_{14} + (y_R \cdot \Delta x_p)m_{15} + (z_R \cdot \Delta x_p)m_{16} + (\Delta x_p)m_{17} = -1$$

$$(x_R)m_{21} + (y_R)m_{22} + (z_R)m_{23} + (x_R \cdot \Delta y_p)m_{24} + (y_R \cdot \Delta y_p)m_{25} + (z_R \cdot \Delta y_p)m_{26} + (\Delta y_p)m_{27} = -1$$

With point samples and their images, the unknown parameters in the equations can be solved out. Based on these parameters, the intrinsic scale factors and the external position and orientation of the camera frame relative to the selected reference coordinate system, denoted by r_x/r_z , r_y/r_z, three Euler angles A_x, A_y, A_z, three translations d_x, d_y, d_z, can be expressed as follows:

$$A_x^{RC} = 180.0 + \arctan(-m_{15}/m_{16})$$

$$A_y^{RC} = \arctan(\cos(A_x^{RC}) \cdot m_{14}/m_{16}$$

$$A_z^{RC} = \arctan((m_{12}/m_{11} - \sin(A_x^{RC}) \cdot \tan(A_y^{RC})) \cdot \cos(A_y^{RC})/\cos(A_x^{RC}))$$

$$d_x^{RC} = (\sin(A_x^{RC}) \cdot \sin(A_y^{RC}) \cdot \cos(A_z^{RC}) + \cos(A_x^{RC}) \cdot \sin(A_x^{RC}))/m_{12}$$

$$d_y^{RC} = -\cos(A_y^{RC})\sin(A_z^{RC})/m_{21}$$

$$d_z^{RC} = m_{17} \cdot d_z^{RC}/(r_x/r_z)$$

$$r_x/r_z = m_{16} \cdot d_x^{RC}/(\cos(A_x^{RC}) \cdot \cos(A_y^{RC}))$$

$$r_y/r_z = m_{26} \cdot d_y^{RC}/(\cos(A_x^{RC}) \cdot \cos(A_y^{RC}))$$

$$r_x/r_y = m_{16} \cdot d_x^{RC}/m_{26} \cdot d_x^{RC}$$

4. EXPERIMENTS AND ACCURACY IMPROVEMENT

4.1. Factors To Influence Calibration Accuracy

There are mainly three sources causing errors in calibration results. They are : the nonlinearness of the structured light plane, camera lens distortion and image quantilizing error, and the limit of the precision of the 3D micrometer stage system. The structured plane light source consists a He-Ne laser and a lens system; it has a good linearness(plat and good focusing) in a certain range[19]. The camera in our experiments is PULNiX-TM560, with a f=25mm lens whose distortion is less than three per cent within the FOV. And the 3D stage employed in our laboratary is a MICRO-CONTROLLE one, which meets an accuracy of 0.01mm.

4.2. Collecting Samples

To calibrate the unknown parameters in 3D vision systems, samples are required, i.e. we need a group of known pairs of spatial points and their corresponding image points. In our calibration experiments, an model with a special designed shape has been used to provide samples. The model is precisely mounted on the 3D micrometer stage, whose position in 3D space can be adjusted by controlling the stage. A global accuracy of 0.02mm in the 3D positions of spatial sample points is reached finally because of the accuracy of the model itself and some mounting errors . Due to the special shape of the model, it is easy to locate the sample points on the image frame by analysising the image pattern of the intersection of the plane light with the model surface. In our case, the digital image abtained after quantilizatioon is 512X512. To improve the precision of locating image points, sub-pixel technique is adapted and the sub- scale is 0.2 pixel. Because the stage is controlled by computer, this procedure to set samples is full automatic and can be done in real time. [19]

4.3 Least Squares [18] [19]

With samples, we can get calibration results based on the calibration equations in section 3. 2. In order to improve accuracy redundant samples are required, so that least square method has to be employed. By considering an error model, here the weighted least square method is selected. For a calibration equation, there are two steps to reach the final result. For instance, the calibration equations in section 3.2.1 can be denoted as [Pi] [Ri]=[Si] or F(L,Ri)=0, where [Ri]=(ni1, ni2, .. , ni5)′ is a vector of calibrated parameters, i=1, 2, [L]=[(xR, yR)j]=g([R1], [R2]) is a vector of the world coordinates of sample points, j=1, 2, ... M, then :

Step I. the estimation of [Ri], [Ri*] can be gotten as:
$[Ri^*]=([Pi]'[Pi])^{-1}[P]'[Si]$, i=1,2 ;

step II. let $[L^*]=g([R1^*],[R2^*])$, $[A]=\partial[F]/\partial[L]\,|\,[L^*],[R^*]$,
$[Bi]=\partial[F]/\partial[Ri]\,|\,[L^*],[R^*]$, $[f0i]=-(F([L^*],[Ri^*])+[A]\cdot([L]-[L^*]))$,
from the equation $[A][V]+[Bi][dRi]=0$:
$[dRi]=([Bi]'\cdot([A][Q][A]')^{-1}[Bi])^{-1}([Bi]'\cdot([A][Q][A]')^{-1}[f0i])$,

where [Q]=[J] [I] [J]′, it can be thought as covariance of [(xR, yR)j], [J] is the matrix of the first partial derivatives of h or the Jacobian, [(xR, yR)j]=h([(xF, yF)j]). Identity matrix [I] is the covariance of [(xF, yF)j], it means that the image coordinates are uncorrelated and the error model is with variances of one pixel, the final calibration results [Ri]=[Ri*]+[dRi], i=1,2, j=1,2,...,M.

4.4. Experiment Results

The proposed algorithm has been used in calibrating several 3D vision systems developed in our laboratary and made a good performance. In our technical 3D coordinates measurement tests, it meets the following accuracy: (unit: mm)

Errors	x coordinate	y coordinate	z coordinate
Average	0.0602	0.0510	0.0694
Maximum	0.1037	0.1121	0.1056

The total range of x, z are 25.00mm and 200. 00mm, respectively. The total range of y(depth) is 20.00mm, the distance from camera is about 400mm.

Appendix I The Proof and Evolvement of the Foundamental Theorem of 2D Perspectivity

[Proof]

The perspective center is point G; the planes are π1 and π2; the selected 2D projective coordinates systems are [O1] and [O2], see Fig.4. G ∉ π1, π2.

Auxiliary coordinate systems [O1′] and [Og] are setted up, there are : O1′ is the intersection of a line and the plane π1, the line passes point G and is vertical to plane π1; $\vec{x1'} // \vec{x1}$, $\vec{y1'} // \vec{y1}$; Og is just located on G; $\vec{Xg} // \vec{X1}$, $\vec{Yg} // \vec{Y1}$, $\vec{Zg} // \vec{Xg} \times \vec{Yg}$. We add a Z2 axis to [O2] and $\vec{Z2} // \vec{X2} \times \vec{Y2}$.

It is known that:

$$p1 \begin{bmatrix} X1 \\ Y1 \\ h1 \end{bmatrix}_{\pi 1} = \begin{bmatrix} 1 & 0 & Lx \\ 0 & 1 & Ly \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X1' \\ Y1' \\ h1' \end{bmatrix}_{\pi 1} = [M1] \begin{bmatrix} X1' \\ Y1' \\ h1' \end{bmatrix}_{\pi 1}$$

where Lx, Ly are translations between [O1] and [O1′].

There is perspective transformation:

$$p2 \begin{bmatrix} X1' \\ Y1' \\ h1' \end{bmatrix}_{\pi 1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/Lz & 0 \end{bmatrix} \begin{bmatrix} Xg \\ Yg \\ Zg \\ h \end{bmatrix} = [M2] \begin{bmatrix} Xg \\ Yg \\ Zg \\ hg \end{bmatrix}$$

where Lz is the distance from point G to plane π1, i.e. $||\overline{GO1'}||$.

Between [Og] and [O2], there is homogeneous transformation:

$$p3 \begin{bmatrix} Xg \\ Yg \\ Zg \\ hg \end{bmatrix} = \begin{bmatrix} ax & bx & cx & dx \\ ay & by & cy & dy \\ az & bz & cz & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X2 \\ Y2 \\ Z2 \\ h2 \end{bmatrix} = [M3] \begin{bmatrix} X2 \\ Y2 \\ Z2 \\ h2 \end{bmatrix}$$

Since every point on plane π2 has the coordinate Z2 equal to zero, it leads:

$$\begin{bmatrix} X2 \\ Y2 \\ Z2 \\ h2 \end{bmatrix}_{\pi 2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X2 \\ Y2 \\ h2 \end{bmatrix}_{\pi 2} = [M4] \begin{bmatrix} X2 \\ Y2 \\ h2 \end{bmatrix}_{\pi 2}$$

Combining the above relations, we have :

$$p \begin{bmatrix} X1 \\ Y1 \\ h1 \end{bmatrix}_{\pi 1} = \prod_{i=1}^{4} [Mi] \begin{bmatrix} X2 \\ Y2 \\ h2 \end{bmatrix}_{\pi 2} = [\psi] \begin{bmatrix} X2 \\ Y2 \\ h2 \end{bmatrix}_{\pi 2} \quad (*)$$

It can be known that :

$$[M2][M3][M4] = \begin{bmatrix} ax & bx & dx \\ ay & by & dy \\ az/Lz & bz/Lz & dz/Lz \end{bmatrix} = [\psi 1]$$

It is apparent that the matrix [ψ1] has the same rank with the matrix $[\psi 1']$,

$$[\psi 1'] = \begin{bmatrix} ax & bx & dx \\ ay & by & dy \\ az & bz & dz \end{bmatrix}$$

Let's consider the following equation:

$$k1 \begin{bmatrix} ax \\ ay \\ az \end{bmatrix} + k2 \begin{bmatrix} bx \\ by \\ bz \end{bmatrix} + k3 \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} .$$

Because homogeneous transformation can assure that the first two colomn being linear independent, so if the $[\psi 1']$ has a rank<3, there must be a group of $k1'$, $k2'$ and $k3'$ and $k3' \neq 0$ to satisfy the above equation. Without losing generality, we can assume $k3'=1$ and there are:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}_{[Og]} = \begin{bmatrix} ax & bx & cx & dx \\ ay & by & cy & dy \\ az & bz & cz & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} k1' \\ k2' \\ 0 \\ 1 \end{bmatrix}_{[O2]}$$

This equation shows that the origin (i.e. the point G) of [Og] has the coordinates ($k1'$, $k2'$, 0, 1) in [O2]; it means that the point G is located on the plane $\pi 2$. But this conclusion is conflict with the hypothesis. Hence, $[\psi 1']$ has a rank=3. It leads that $[\psi 1]$ has a rank=3 . And [M1] has a rank=3, so $[\psi]$ has also a rank =3 because of the relation: $|\psi| = |M1| \cdot |\psi 1|$.

The conclusion that $[\psi]$ has a rank=3 shows that a 2D perspectivity is a 2D projectivity. Considering that there are nine elements in $[\psi]$, but there are only eight independent ones because the parameter p is free. For (*), three pairs of samples can only produce nine equations, they can not exactly determine the twelve unknown variables, including eight elements in $[\psi]$ and the three p

corresponding to different sample pairs. So , more than three pairs of samples are needed to exactly determine a 2D perspectivity. Based on the foundamental theorem of 2D projectivity , finally we reach the conclusion that a pair of corresponding quadrangular sets in a specified order can exactly determine a perspectivity.

The End

[Evolvement]

The case in above proof is that the selected projective coordinate systems are two 2D ones located in the two planes. In the following discussion, we consider that one of the selected coordinate systems is a 3D one [Or] which has a random position and random orientation. Namely, this model is more suitable for a practical 3D vision system. See Fig. 1.

Similar to the case in the proof , again there is :

$$p \begin{bmatrix} X1 \\ Y1 \\ h1 \end{bmatrix}_{\pi 1} = [M1]\,[M2]\,[M3] \begin{bmatrix} Xr \\ Yr \\ Zr \\ hr \end{bmatrix} .$$

From the equation $Zr = Kx.Xr + Ky.Yr + Ko$, of the plane $\pi 2$ in [Or], there is:

$$\begin{bmatrix} Xr \\ Yr \\ Zr \\ hr \end{bmatrix}_{\pi 2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Kx & Ky & Ko \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Xr \\ Yr \\ hr \end{bmatrix}_{\pi 2} = [M4'] \begin{bmatrix} Xr \\ Yr \\ hr \end{bmatrix}_{\pi 2}$$

Again, it can be gotten that:

$$p \begin{bmatrix} X1 \\ Y1 \\ h1 \end{bmatrix}_{\pi 1} = [\psi'] \begin{bmatrix} Xr \\ Yr \\ hr \end{bmatrix}_{\pi 2} .$$

Repeat the similar analysis in the proof, it will be known that $[\psi']$ has also a rank=3 and the same conclusion can be reached.

The End

Appendix II The Definitions of the Elements in Homogeneous Matrix

$a_x = \cos A_y . \cos A_z$

$a_y = -\cos A_y . \sin A_z$

$a_z = \sin A_y$

$b_x = \sin A_x . \sin A_y . \cos A_z + \cos A_x . \sin A_z$

$b_y = -\sin A_x . \sin A_y . \sin A_z + \cos A_x . \cos A_z$

$b_z = -\sin A_x . \cos A_y$

$c_x = -\cos A_x . \sin A_y . \cos A_z + \sin A_x . \sin A_z$

$c_y = \cos A_x . \sin A_y . \sin A_z + \sin A_x . \cos A_z$

$c_z = \cos A_x . \cos A_y$

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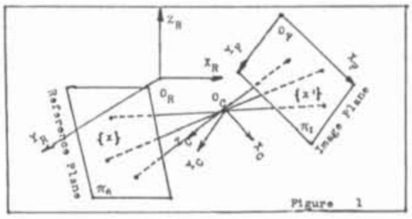

Figure 1

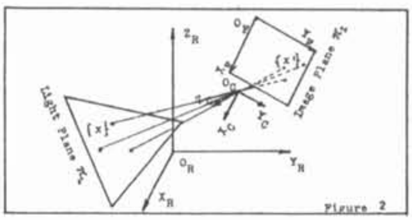

Figure 2

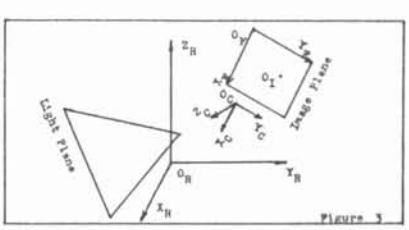

Figure 3

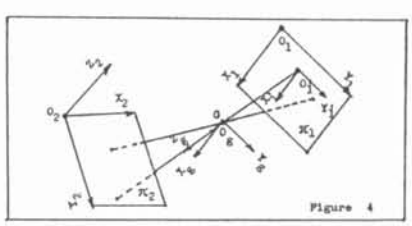

Figure 4