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Modeling the Stone Floor Based on Excavation Information Using Implicit Polynomial

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Abstract

High-quality modeling method is required for the virtual reconstruction of ruins using mixed reality. Stone floor is one of the important parts of the ruins.In order to render the stone floor realistically, it requires a high-quality model produced by excavation information. For the reconstruction of the lacked part of stone floor, shape modifying method while maintaining a shape characteristic and the distribution of the original is required. In this paper, we propose a shape-forming method to reconstruct the stone floor from excavated data by using Implicit Polynomial(IP). IP which is the implicit function curved surface can perform interpolation and blending easily.

Figure 1. Virtual reconstruction of the ruin

1 Introduction

Recently, the mixed reality has been focused to exhibit the virtual reconstruction of ruins. It can superimpose the archaeological building in ancient times on current landscape using Head Mounted Display or smart phones. Fig.1 shows one of the virtual reconstructions of ruins. To show buildings in ancient times as mixed reality, buildings are reconstructed by CGs based on the archaeological evidence. However, the stone floor that is ground in Fig. 1 does not have reality, because of the simple texture.

Most of ruins lost their own parts because of aging or disaster to name a few. Therefore, supplementing the lost part as CG has a big role to show the ancient scene in the present time. To show more accurate scenes, not only reproduction of building but also the reconstruction of the background, such as stone floor, is important

In this paper, we propose a shape-forming method to reconstruct the stone floor excavated data by Implicit Polynomial(IP). We reconstruct the stone floor from shapes of each stones and excavation information like gap among stones. Depending on gaps, we determine the paved degree and transform the shape of the original stone smoothly. We achieve the smooth interpolation between original stones area and shortage area by characteristics of the IP. We express natural distribution using Laguerre Voronoi-diagram to control stone shortage area. Furthermore, our method can handle not only 2D model but also 3D model because of the feature of the IP. In this paper, we focus on discussing 2D modeling.

2 Related works

In civil engineering and computer graphics fields, the technique to generate the stone-distribution model was proposed. Miyata et al.[1] achieved this by the closest packing of square particles and rearranged the shape of each particle. In this method, unnatural gap and heap occurred because the shape and the position of stones are set independently.

We have three advantages to use a Voronoi-diagram to control the distribution among stones.

1. Center gravity of stones are usable to Voronoi tessellation.

2. The shape of each Voronoi regions is similar to stone shapes because each regions are convex polygon.

3. We can adjust gaps later.

Table 1. Comparison between related works and proposed method

	Peytavie et al.	Mollon et al.	Proposed
3D	Possible	Impossible	Possible
Voronoi Diagram	Multiplicatively Weighted Voronoi	Voronoi	Laguerre Voronoi
Shape Fitting	Erosion	Fourier Descriptor	IP

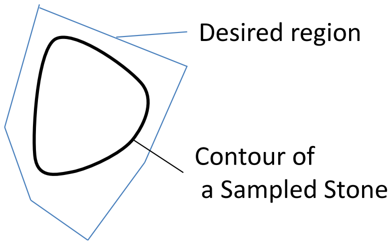

Figure 2. Issue of IP Fitting

Peytavie et al.[2] proposed a method to generate stonework using the multiplicatively weighted Voronoi-diagram. Eq.(1) shows Voronoi diagram.

$$R(S; P_i) = \{P \in R^d \mid l(P, P_i) < l(P, P_j), j \neq i\} \quad (1)$$

$l(P,Q)$ is distance between point P and point Q. $S = \{P1, P2, ..., P_n\}$ is set of the generatrices in the space R^d. $R(S; P_i)$ is Voronoi-diagram of generatirices P_i. It is class of P which is nearest point in S. In contrast the distance of the multiplicatively weighted Voronoi-diagram is shown Eq.(2).

$$l(P, P_i) = \frac{1}{g_i}\{(x - x_i)^2 + (y - y_i)^2\} \quad (2)$$

It can regulate the size of the stone by g.

Mollon .e.g.[3] studies the generation of a model spreading powder and granular material such as the sand. It represents the contour by Fourier descriptor. It is used for the setting of the shape and gap control of the volonoi-diagram.

Comparison between related works and our proposed method is shown in Table 1. Petavie's method can generate 3D model, but is difficult to modify the shape of stone to desired regions. Mollon's method can generate the desired stone shape, but is not suitable for distribution of 3D model. Our proposed method realizes generation of 3D model and free transformation using IP and Laguerre Voronoi-diagram.

3 IP Fitting

In order to naturally represent the stone excavation information, in this paper we propose to use an implicit polynomial (IP) to interpolate two shape contours where the one contains the other one (Fig. 2).

3.1 Implicit Polynomial(IP)

IP is an implicit function defined by the polynomial of degree n

$$f(\mathbf{x}) = \sum_{0 \leq i,j,k,i+j+k \leq n} a_{ijk} x^i y^j z^k = 0, \quad (3)$$

where a_{ijk} is the coefficient of polynomial f. The surface of an object can be represented by $f(\mathbf{x}) = 0$. For example, an unit sphere can be represented by a quartic as:

$$f(\mathbf{x}) = -1 + x^2 + y^2 + z^2 = 0. \quad (4)$$

$f(x)$ can be rewritten in the inner-product form of two vectors:

$$f(\mathbf{x}) = \underbrace{\begin{pmatrix} 1 & x & \cdots & z^n \end{pmatrix}}_{\mathbf{m}(\mathbf{x})^T} \underbrace{\begin{pmatrix} a_{000} & a_{100} & \cdots & a_{00n} \end{pmatrix}^T}_{\mathbf{a}}, \quad (5)$$

where $m(\mathbf{x})$ is monomial vector and a is coefficient vector. Given the point cloud on object surface $\{\mathbf{x}_i\}_{i=1}^{l}$, IP fitting algorithm finds the best coefficients that satisfy $f(\mathbf{x}) = 0$, i.e., it minimizes the distance between point cloud and zero set of the IP w.r.t. coefficients. This problem can be formulated as a linear equation system:

$$\mathbf{Ma} = \mathbf{b}, \quad (6)$$

where $M = \begin{pmatrix} \mathbf{m}(\mathbf{x}_1) & \mathbf{m}(\mathbf{x}_2) & \cdots & \mathbf{m}(\mathbf{x}_l) \end{pmatrix}^T$,and $\mathbf{b}$ is a zero vector. Since Eq.6 is an over-determined linear system, it can be solved by least squares method: $a = (M^T M)^-1 M^T b$. To avoid the zero solution, we adopt 3L method [4] that introduces two linear constraints by generating two extra parallel layers around the original point cloud: $f(\mathbf{x}_+) = +e$ and $f(\mathbf{x}_-) = -e$, where $\mathbf{x}_+$ and $\mathbf{x}_-$ are the points on outer and inner layers respectively and e is the distance between the extra layers and original surface.

However, the singularity of $M^T M$ often frustrate the numerical stability of the solution of Eq.6, fortunately

$$\left(M^T M + \kappa D\right)\mathbf{a} = M^T \mathbf{b} \quad (7)$$

is developed to improve the singularity of matrix $M^T M$ by modifying the diagonal elements. Here, κ is a positive number which is called RR parameter and D is a diagonal matrix. Also, to overcome the over fitting problem, we adopt an adaptive fitting approach proposed in [5] that adaptively determines the degree of an IP according to the shape complexity.

3.2 Shape Interpolation by IP Coefficients

As shown in Fig. 2, 2 types of IPs are chosen to fit the contour of each stone and the contour of desired region respectively. Then the interpolation is carried out between the IP fits of stone and desired region. That is, the interpolation can be viewed as the linear combination between two coefficient vectors of the IPs. Suppose the interpolated coefficient vector of IP is denoted by $\mathbf{a}_{lerp}$, stone contour $\mathbf{a}_{stone}$ and desired region $\mathbf{a}_{voronoi}$, then

$$\mathbf{a}_{\text{lerp}} \quad = \quad \alpha \mathbf{a}_{\text{voronoi}} + (1 - \alpha)\mathbf{a}_{\text{stone}} \quad (8)$$

4 Distribution by Laguerre Voronoi Diagram

An Voronoi diagram which used Laguerre distance is called Laguerre Voronoi diagram. Laguerre distance can be defined as:

$$l(P, P_i) = (x - x_i)^2 + (y - y_i)^2 - r_i^2 \quad (9)$$

Figure 3. Input Image

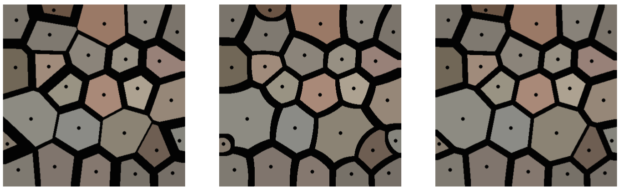

Figure 4. Area fitting on Voronoi diagram (a)Voronoi diagram(b)Multiplicatively weighted Voronoi diagram(c)Laguerre Voronoi diagram

where $P_i(x_i; y_i)$ is called generatrix with a non-negative real number r_i. For an arbitrary point $P(x; y)$, its Laguerre distance to P_i is calculated by $l(P, P_i)$. This distance can be viewed as the length of the tangent drawn in a circle of the radius r_i, and used as an index which compares the distance from a circle.

In the coordinate system $P_i(1; 0), P_j(-1; 0)$, boundary of the Laguerre Voronoi region is $x = (r_j^2 - r_i^2)/4$. Therefore, a straight line perpendicular to the line segment which connects P_i and P_j serves as a boundary, and area of the region is directly proportional to r. Since each region serves as a convex polygon and a size can be controlled. It is suitable for generation of stone floor.

We confirm the advantage of the Laguerre Voronoi diagram for stone floor based on comparison with other Voronoi method. We assume the centroid of the stone as generatrix of Voronoi diagram. g of multiplicatively weighted Voronoi diagram and r^2 of Lagguerre Voronoi diagram are defined $g = r^2 = S_o/\pi$, S_o is the area of the original stone. Area of the generated stone is S_G, common area with original stone S_O and S_G is $S_{O\cap G}$. If S_G is larger than S_O, S_G can be adjusted by the erosion until $S_G \leq S_O$. Here we introduce 2 evaluation factors: A, Capability of $S_O = S_G$ and B the mean value of $(S_{O\cap G}/S_O)$. The former one checks the controllability of the area, and the latter one checks the controllability of the shape. As an example, the input of the stone floor is shown in Fig. 3. Desired region is arbitrary shape. We try to use Voronoi region from picture of a stone floor as desired region.

In Fig. 4, (a) shows the reconstruction result using Voronoi diagram, (b) shows the Voronoi diagram using Multiplicatively weights, and (c) shows the diagram using Laguerre Voronoi. The value of the evaluation factor A and B is shown in Table 2. As result, Voronoi diagram shows bad performance in the 3 methods, and multiplicatively weighted Voronoi diagram and Laguerre Voronoi diagram get almost same performance. However the reconstruction result of Multiplicatively weighted Voronoi diagram has some unnatural shape especially on big stone (see Fig. 4).

Table 2. Evaluation of Voronoi diagram

Voronoi types	$S_O = S_G$ ratio	$(S_{O\cap G}/S_O)$ mean
Voronoi diagram	92%	83.8%
Multiplicatively weighted Voronoi diagram	100%	86.5%
Laguerre Voronoi diagram	100%	86.4%

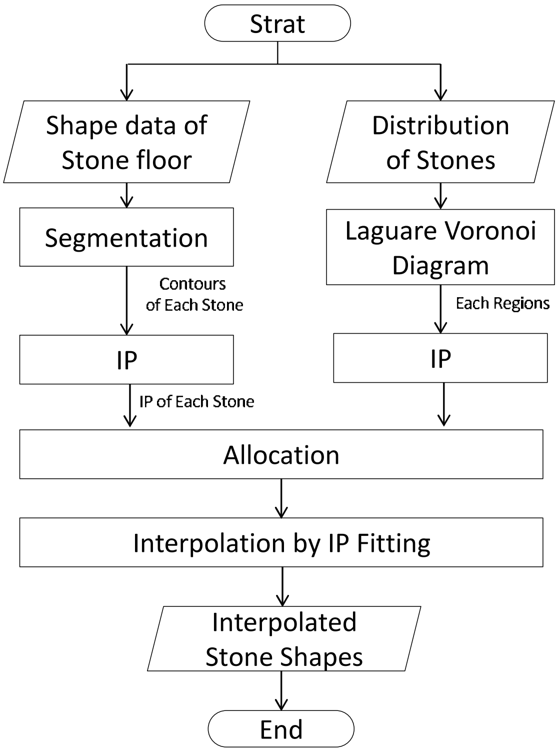

Figure 5. Flow chart

5 Experiment

5.1 Process flow

In this section, we confirm the effectiveness of our method. Fig.?? shows the flowchart of this experiment. From Start, two data streams are derived. Left process is to calculate IP from shape data of stone floor. The segmentation process calculates contours of stones by extracting stones from stone floor data. Right process is for IP calculation from ideal distribution of stones. Laguerre Voronoi diagram determine desired region of each stone for interpolation. IP of each stones is calculated by IP fitting process from contours of each stone region. The pieces of segmented stone is placed to the region of the Voronoi diagram. Blending process interpolate regions by IP.

Adjusting the pattern of the distribution based on excavation information, we can edit the stone floor easily because we can set Voronoi-diagram arbitrarily and independently.

5.2 Result

Fig.5 (L) and (R) shows regions of a stone floor, and the desired regions respectively. The desired re-

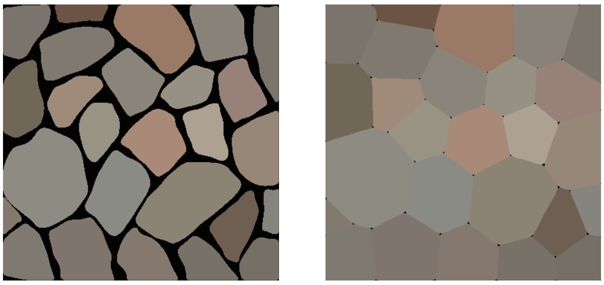

Figure 6. (Left)Shape data of Stone floor, (Right)Distribution of Stones

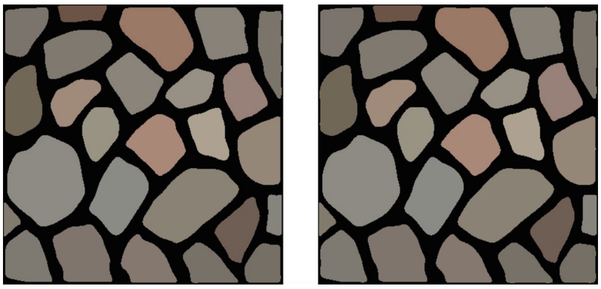

Figure 7. α=0.0, κ=L)0.001,R)0.01

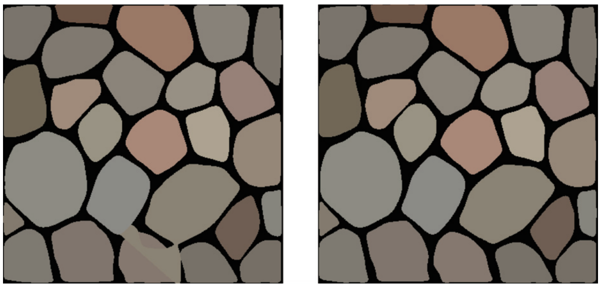

Figure 8. α=0.5, κ=L)0.001,R)0.01

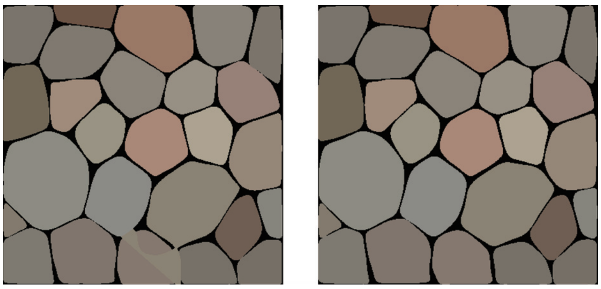

Figure 9. α=0.8, κ=L)0.001,R)0.01

gions are distribution of stones generated by Laguerre Voronoi Diagram.

We calculated the IP contour of each stone and that of each desired regions by IP Fitting. Using IP, each contours are expressed by $f(x) = 0$. We calculated the smooth transformation among two regions by linear interpolation of coefficients which is same degrees of IP. We applied the 3L method and ridge regression analysis for Adaptive IP Fitting. We set scale adjustment as 0.9 times as the initial shape of the stones because the size of stone regions are smaller than Voronoi regions.

Fig.6-8 show the transformation results. In Fig.6-8, we set parameter α as 0.0, 0.5 and 0.8 respectively. In left side images, we set the parameter κ as 0.001. In right side images, we set κ as 0.01. Setting κ as 0.001, some stone shapes could not reconstitute by linear interpolation. When κ is equal to 0.01, all stones reconstitute its shapes well, and gaps among each stones are adjustable. This result shows the effectiveness of our method to transform the stone floor well.

5.3 Discussion

When we set $\kappa = 0.001$, $\alpha = 0.5$ or $\alpha = 0.8$, the interpolation error occurred. In addition, using IP, reconstructed contours of stones were slightly rounded off in every situation. It is because a property of the IP is difficult to express the shape which is not smooth. Therefore, it is effective to set larger κ for stable IP Fitting. However, there is a trade-off between shape precision and stability. The failure judgment can be calculated by the ratio of each convex area. It is effective to minimize κ using this failure judgment.

6 Summary

We proposed a method for smooth interpolation of stone floor by IP and Laguerre Voronoi Diagram. In experiments, we confirmed that the gap is arbitrarily adjustable. In future work, we develop the editing tool to handle nor only contours but also detail shapes such as textures.

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