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An Adaption of the Lucy-Richardson Deconvolution Algorithm to Noncentral Chi-Square Distributed Data

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Abstract

The Lucy-Richardson Algorithm is a well-known iterative method for the deconvolution of images convolved with a known point spread function. It is derived from a statistical point of view as it converges to the maximum-likelihood solution under the condition that the data follow a Poisson distribution. This assumption holds true for images detected by a digital camera. However, there are images not following a Poisson but rather a noncentral chi-square distribution. Here we show an adaption of the Lucy-Richardson algorithm to be used for data following this probability distribution. Its application to simulated and real data from an imaging radar sensor shows its advantage over the original algorithm.

1 Introduction

Images recorded by an arbitrary device are blurred by the influence of a non-perfect recording device. This process can be modeled as the convolution of the unblurred image with a known or unknown point spread function (PSF). E. g., [1] used a convolutional model for the diffraction of images acquired by a cryogenic infrared focal array. The authors derived the PSF as a combination of an atmosphere transfer function, an optical transfer function and a detector transfer function. [2] modeled the blurring of barcode images by camera shake by convolution as well, but without exact knowledge of the PSF. Beyond that, the detection of objects by a scanning radar sensor can be described as the convolution of the objects with the radar antenna beam [3].

Radar sensors have been used in vehicles for over a decade in order to implement several driver assistance systems. E. g., adaptive cruise control needs information about the distance and the relative speed of preceding vehicles, which both can be measured by a radar sensor very accurately. Future driver assistance systems avoiding or mitigating crashes will also need information about the width of vehicles ahead in order to tell whether an evasion by the driver is still possible or a crash is unavoidable. As the installation space in a car is very limited to a radar, the antenna aperture D is limited as well, leading to a wide antenna beam

$$B = \operatorname{sinc}^2\left(\frac{\phi D}{\lambda}\right) \tag{1}$$

with the radar wavelength λ and the azimuth angle ϕ. The convolution of objects with the wide antenna beam makes the width and the lateral position of other vehicles hard to determine (see Figure 1).

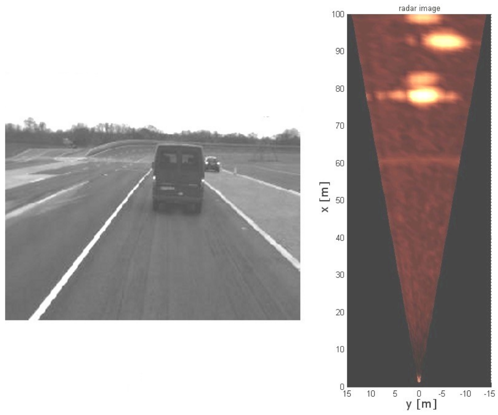

Figure 1. Car in the image of an automotive scanning radar sensor.

As convolution tends to obscure important information, so-called deconvolution methods try to reverse the convolution. Practical examples can be found in [4]. Although deconvolution often achieves good results, an omnipresent problem coming with it is noise amplification. A method among many others trying to cope with it is shown in [5]. The autors combined Wiener adaptive filtering with the Lucy-Richardson algorithm and restored an image of a barcode taken by the camera of a mobile phone. [6] combined dual-tree complex wavelet denoising with Lucy-Richardson deconvolution. [7] modified the iteration step so that it does not cause changes in image areas with predominant noise. [8] proposed a combination of the Lucy-Richardson algorithm with a regularizing constraint based on total variation leading to an improved deconvolution result.
Here, we propose a non-heuristic adaption of the Lucy-Richardson algorithm optimized for noncentral chi-square distributed data. In section 2, we describe the original Lucy-Richardson algorithm. We then present an adaption of the algorithm to noncentral chi-square distributed data. Afterwards we test the adaption on simulated (section 4) and on real data from an imaging radar sensor (section 5). We end with a brief conclusion (section 6).

2 The Lucy-Richardson algorithm

The Lucy-Richardson deconvolution is an iterative algorithm optimized for Poisson distributed data [9], [10]. Poisson distribution can be assumed in the case that an image is recorded by a digital camera [11].

In the absence of noise, a blurred image $\bar{i}(x)$ is formed from an unblurred image $o(x')$ by the convolution

$$\bar{i}(x) = \int o(x')s(x,x')dx' \tag{2}$$

with the PSF $s(x,x')$ [4]. In the presence of Poisson noise, the probability density function (PDF) of the measured intensity of a noisy pixel $i(x)$ around its mean $\bar{i}(x)$ is

$$P(i|\bar{i}) = \frac{\bar{i}^{i}}{i!}e^{-\bar{i}}. \tag{3}$$

The log likelihood of an entire set of data values is

$$\ln L = \int i(x)\ln\bar{i}(x) - \bar{i}(x) - \ln\left(i(x)!\right)dx. \tag{4}$$

To find the maximum-likelihood solution, we take the derivative with respect to o and require it to be zero:

$$\frac{\partial \ln L}{\partial o(x')} = \int \left[\frac{i(x)}{\bar{i}(x)} - 1\right] s(x,x')dx \stackrel{!}{=} 0. \tag{5}$$

In this case, the iteration factor of the Lucy-Richardson algorithm

$$\hat{o}^{(k+1)}(x') = \hat{o}^{(k)}(x')\frac{\int\left[\frac{i(x)}{\hat{i}^{(k)}(x)}\right]s(x,x')dx}{\int s(x,x')dx} \tag{6}$$

is unity [4] with

$$\hat{i}^{(k)}(x) = \int \hat{o}^{(k)}(x')s(x,x')dx' \tag{7}$$

where $\hat{o}^{(k)}$ is the k^{th} estimate of the unblurred image. So if the algorithm converges, it converges against the maximum-likelihood solution so that $\hat{i} \rightarrow \bar{i}$.

3 Adaption to noncentral chi-square distributed data

The assumption of Poisson distributed data does not hold true for all kinds of images. Images from radar sensors consist of values representing the backscattered power from each range and azimuth cell. The distribution of the power of radar signals can be modeled as a noncentral chi-square distribution [12], [13]. This distribution with a degree of freedom n arises if the squares of n normally distributed random variables are summed up. In the case of a noncentral chi-square distribution with two degrees of freedom, equation 3 has to be

$$P(i|\bar{i}) = \frac{1}{2\sigma^2}e^{\left(\frac{-(i(x)+\bar{i}(x))}{2\sigma^2}\right)}I_0(a) \tag{8}$$

with

$$a = \frac{\sqrt{i(x)\bar{i}(x)}}{\sigma^2} \tag{9}$$

where I_0 is the modified Bessel function of the first kind with order zero. Note that in this case $\bar{i}(x)$ is not the mean of the noncentral chi-square distribution, nevertheless it depends on the means of the underlying normal distributions:

$$\bar{i} = \mu_S^2 + \mu_N^2. \tag{10}$$

Here we assume the case of a resulting value $\bar{i}$ consisting of a signal part with the mean voltage μ_S and a noise part with the mean voltage μ_N. Under the assumption of a mean noise voltage of zero, $\bar{i}$ is the power of the signal part, which is the value of interest here.

The log likelihood (equation 4) is

$$\ln L = \int \ln\frac{1}{2\sigma^2} - \frac{i(x)+\bar{i}(x)}{2\sigma^2} + \ln I_0(a)\,dx. \tag{11}$$

Taking the derivative with respect to o and requiring it to be zero in analogy to equation 5 leads to

$$\frac{\partial \ln L}{\partial o(x')} = \int -\frac{s(x,x')}{2\sigma^2} + \frac{I_1(a)}{I_0(a)}\frac{\sqrt{i(x)}s(x,x')}{2\sigma^2\sqrt{\bar{i}(x)}}dx \stackrel{!}{=} 0. \tag{12}$$

Then, the iteration step according to equation 6 is

$$\hat{o}^{(k+1)}(x') = \hat{o}^{(k)}(x')\frac{\int\frac{I_1(\hat{a})}{I_0(\hat{a})}\sqrt{i(x)}s(x,x')dx}{\int s(x,x')\sqrt{\hat{i}^{(k)}(x)}dx} \tag{13}$$

with

$$\hat{a} = \frac{\sqrt{i(x)\hat{i}^{(k)}(x)}}{\sigma^2}. \tag{14}$$

This modification of the iteration factor adapts the algorithm to noncentrally chi-square distributed data.

4 Simulation results

In order to make statements about the benefit of the algorithm, its performance is tested on simulated convolution data. For this purpose, an unblurred image function is convolved with an antenna diagram (PSF, see Figure 2) calculated from equation 1. Noncentrally chi-square distributed noise is added to the result (see Figure 3). The noisy signal is then deconvolved by the adapted Lucy-Richardson algorithm. In this connection, the relaxation method proposed in [8] is included. The result is shown in Figure 4 compared to the result from an original Lucy-Richardson algorithm. Especially in low signal-to-noise ratio regions the chi-square adapted algorithm has the edge over its original form. Figure 5 shows the root mean squared error (RMSE) between the deconvolution result and

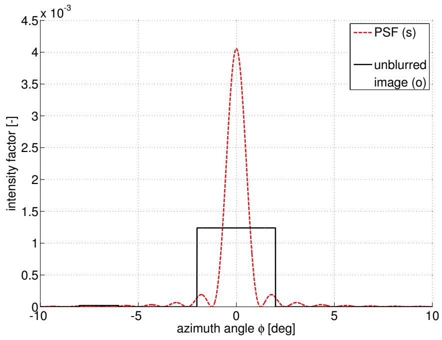

Figure 2. Unblurred image and point spread function (PSF) used for simulation.

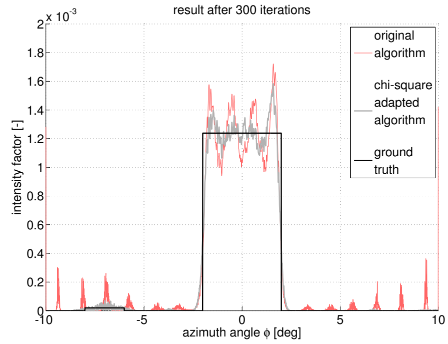

Figure 4. Result from the adapted Lucy-Richardson algorithm compared to the result from its original form. Note the still detectable small target at $-7°$.

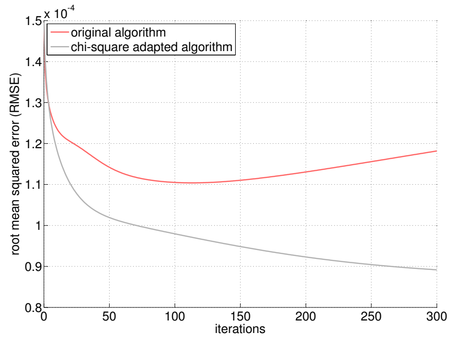

Figure 5. Root mean squared error (RMSE) between the deconvolution result and the unblurred image function (ground truth) as a function of the number of iterations. The adapted Lucy-Richardson algorithm outperforms the original implementation.

the unblurred image function (ground truth) as a function of the number of iterations. The adapted Lucy-Richardson algorithm outperforms its original as there is no tendency to the noise amplification in areas where the unblurred image function is zero.

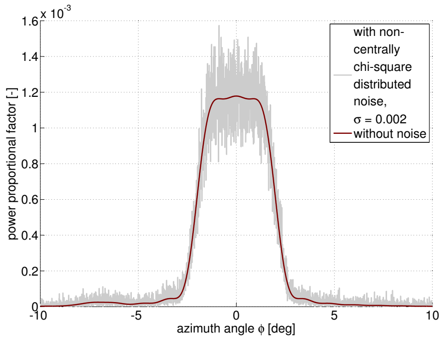

Figure 3. Result of the convolution and the addition of noncentrally chi-square distributed noise.

5 Real data results

In the next step, the performance of the algorithm is examined for real data. For this purpose, radar images from an experimental scanning imaging radar sensor mounted on a vehicle (Figure 7) are used. In regard to the principle of measurement, the sensor is similar to today's mechanically scanning adaptive cruise control radar sensors. The parameter σ is estimated from the distribution of power values in cells without any radar targets (see Figure 6). Consequently no heuristic parameter is needed for the algorithm, making it universally valid in the case of noncentrally chi-square distributed data.

As a common traffic scene, the approach towards a still standing vehicle and a pedestrian standing next to it is examined (Figure 8). Figure 9 shows the relative power data from the radar sensor for the distance of 61 m from the sensor, where the standing vehicle and the pedestrian are situated. Whereas the width of the vehicle is difficult to determine from the original data, it can be seen better after the application of deconvolution. The original Lucy-Richardson algorithm

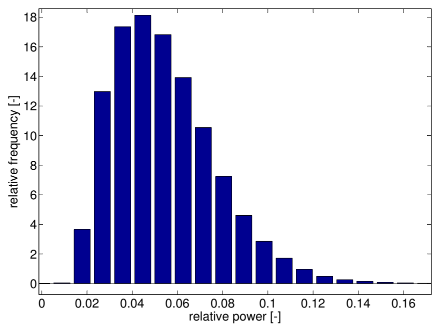

Figure 6. Distribution of power values in cells without any radar targets.

and its chi-square adaption lead to a very similar result in the area of the car and the pedestrian. In the area of empty space, the chi-square adaption annihilates the noise nearly completely, whereas the original implementation does not.

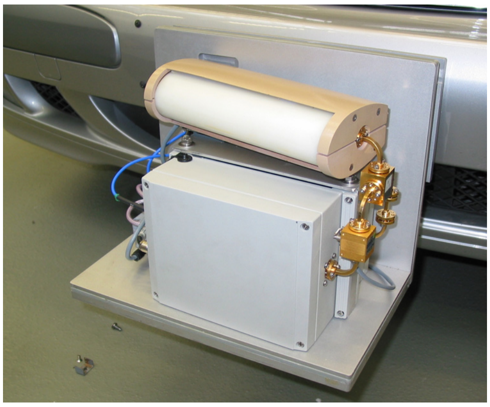

Figure 7. Experimental imaging radar sensor.

Figure 8. Vehicle and pedestrian detected by the experimental imaging radar sensor.

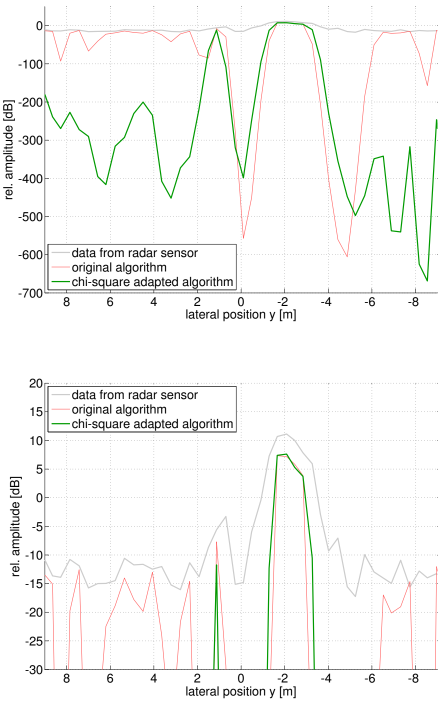

Figure 9. Intensity of the rear of a vehicle and a pedestrian (Figure 8) in a distance of 61 m over the lateral coordinate y (general and detailed view).

6 Conclusion

We have presented a modification of the Lucy-Richardson deconvolution algorithm adapted to noncentrally chi-square distributed data. The Lucy-Richardson algorithm converges to the maximum-likelihood solution under the condition that the underlying data follow a Poisson distribution. In analogy to this derivation, the iteration factor is modified so that the algorithm converges to the maximum-likelihood solution for the case of noncentrally chi-square distributed data. Especially for low signal-to-noise ratios the adapted algorithm outperforms the original one used on simulated data as well as on real data from an automotive imaging radar sensor. Future work can include the combination of this adaption with other Lucy-Richardson adaptions concerning e. g. noise reduction or convergence speed.

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